

Bank Mergers, Competition and Liquidity*

Elena Carletti[†], Philipp Hartmann[‡] and Giancarlo Spagnolo[§]

This draft: November 6, 2002

Abstract

We provide a model of the impact of bank mergers on loan competition, individual reserve management and aggregate liquidity risk. Banks hold reserves against liquidity shocks, refinance in the interbank market and compete in a differentiated loan market. A merger creates an internal money market that induces financial cost advantages and may increase reserve holdings. We assess the liquidity risk and the expected liquidity needs for each bank and for the system, and relate them to the degree of competition in the loan market. Plausible scenarios emerge in which a more competitive environment is beneficial for the liquidity situation of the interbank market.

JEL Classification: D43, G21, G28, L13

Keywords: Credit market competition, bank reserves, internal money market, liquidity risk, interbank markets, banking system liquidity

1 Introduction

The last decade has witnessed a substantial number of mergers and acquisitions in the financial services sector of many industrial countries. This ‘merger movement’ has been documented in detail and generally discussed in various official reports and research papers.¹ For

*We would like to thank Hans Degryse, Fiorella De Fiore, Martin Hellwig, Cornelia Holthausen, Haizhou Huang, Roman Inderst, Andreas Irlen, Simone Manganelli, Martin Ruckes, Rune Stenbacka and Jürgen Weigand for comments and suggestions. Thanks also to the participants of the 2002 European Summer Symposium in Financial Markets in Gerzensee, the 2002 European Finance Association conference in Berlin, the 2002 European Economic Association and Econometric Society European Meetings in Venice, the 2002 German Finance Association meetings in Cologne, the Launching Workshop of the ECB-CFS research network on “Capital Markets and Financial Integration in Europe” in Frankfurt, the Third Joint Central Bank Research Conference on “Risk Measurement and Systemic Risk” in Basel, the CEPR/Bank of Finland conference “Moral Hazard in Banking”. We appreciated the excellent research assistance by Andres Manzanares. Any views expressed are only the authors’ own and do not necessarily coincide with the views of the ECB or the Eurosystem.

[†]University of Mannheim, Department of Economics, 68131 Mannheim, Germany, email: carletti@rumms.uni-mannheim.de.

[‡]European Central Bank, DG Research, Kaiserstrasse 27, 60311 Frankfurt, Germany, and CEPR, email: philipp.hartmann@ecb.int.

[§]University of Mannheim, Department of Economics, 68131 Mannheim, Germany, and CEPR, email: gianca@uni-mannheim.de.

¹See e.g. ECB (2000), OECD (2000), Group of Ten (2001); and Boyd and Graham (1996), Berger et al. (1999), Hanweck and Shull (1999), Huizinga et al. (2001), and Dermine (2000).

example, it was observed that the phenomenon was particularly concentrated among banking firms, that this type of consolidation accelerated during the last years of the 1990s, that most M&As occurred within national borders and that - as a consequence - many countries (e.g. Australia, Belgium, Canada, France, the Netherlands and Sweden) reached a situation of high banking sector concentration or faced a further deterioration of an already previously concentrated sector, whereas a few others (notably Germany and the United States) remained relatively unconcentrated. The origins of the ‘merger movement’ were found, *inter alia*, in technical progress (particularly in communication technology), deregulation, general globalisation and the resulting competitive challenges for financial firms and, related to the latter, monetary integration in Europe. Of particular interest for policy makers, market participants and researchers are the consequences of such an extensive consolidation process for the efficiency and competitiveness of bank intermediation, for market liquidity and financial stability and for the working of monetary policy.

In the present paper we address some of these issues and draw some tentative policy conclusions. We provide a theoretical basis for the joint analysis of the impact of mergers on competition among banks and of their effects on individual reserve management and banking system liquidity. The Ferguson ‘Report on Consolidation in the Financial Sector’ pointed out that ‘...by internalising what had previously been interbank transactions, consolidation could reduce the liquidity of the market for central bank reserves, making it less efficient in reallocating balances across institutions and increasing market volatility’ (Group of Ten, 2001, p. 20). Although the central banks contributing to this report did not see any evidence so far that financial sector consolidation had led to this result, they agreed that the situation should be monitored carefully.

Our main aim is to address the link between bank consolidation and bank liquidity at two levels, the level of individual banks and the level of the banking system as a whole. At the individual level, we are interested in how banks compete in loan markets, how they manage their reserve assets and what liquidity risks they are taking in doing this. We then ask the question in which way mergers change individual bank behaviour, in particular regarding the level of reserves they hold to insure against liquidity risk. At the aggregate level, we are interested in the overall level of loan rates in the system and in the question how individual reserve choices and liquidity risks add up to system wide liquidity fluctuations. In particular, we analyse how bank consolidation affects money market liquidity. Private money market liquidity is important in two respects. First, greater aggregate liquidity fluctuations may make it more difficult for central banks to keep money market rates stable around policy rates. Second, in the absence of a central bank or in the case where central banks cannot perfectly compensate for all liquidity shocks occasional shortages may endanger the stability of financial institutions. In conducting our analysis, we are not only able to address the question whether bank consolidation may drain liquidity from the money market and if yes in which way, but we can also study whether the competitiveness of bank loan markets is complementary or in conflict with money market liquidity. Moreover, our results are suggestive of how central bank liquidity management, the implementation of monetary policy, may have to change in response to the effects of large mergers. This indicates whether bank consolidation increases or reduces the dependence of the banking sector on public liquidity provision. Finally, we use our framework for an exploratory analysis of the bank lending channel in the transmission of monetary policy and how it is affected by merger activity.

To address those issues we develop a model combining liquidity considerations from the banking literature with competition considerations from the industrial organization literature. Our set-up describes banks as raising deposits to invest in long-term loans to entrepreneurs and in liquid short-term assets (reserves). On the market for loans banks compete in prices and retain some market power through differentiation. They hold reserves as a cushion against stochastic liquidity shocks originating from uncertainty about whether depositors wish to withdraw before loans mature. The share of deposits withdrawn early is distributed independently across banks. If depositors' liquidity demand exceeds reserves, a bank can fund the difference by borrowing in the interbank market. The interbank or money market redistributes reserves from banks with excess liquidity to banks with individual liquidity shortages. However, as we assume aggregate uncertainty, the economy-wide demand for liquidity can sometimes exceed the total stock of available reserves. In those circumstances the missing liquidity can be provided by a central bank.

Banks choose reserves balancing the marginal benefit of lower refinancing needs with the marginal cost of having to raise more deposits. At the optimum, reserve holdings increase with the interbank market rate and decrease with the deposit rate. Equilibrium loan rates are set at the level that equates the marginal revenue of providing loans with the marginal costs of monitoring loans, refinancing in the interbank market and raising deposits. They decrease in the competition parameters (number of banks, substitutability of loans) and increase in the cost factors (monitoring costs, interbank and deposit rates).

The occurrence of a merger modifies banks' behaviour concerning both liquidity management and loan market competition. As regards the former, an important feature of our analysis is that mergers can create an internal money market. This form of internal capital market implies cost advantages, as liquidity can be reshuffled without paying the interbank market rate. Interestingly, this internal insurance mechanism rather tends to lead to *higher* reserve holdings. The reason is that as long as the interbank market rate is not too high relative to deposit rates, the positive externality of an additional unit of reserve held in one part of the bank on the other part of the bank dominates the forces of diversification. A low interbank rate means that reserve ratios are relatively low before merger, so that the internalisation of the positive externality is more important and reserves rise after the merger. Only for a very high interbank rate are pre-merger reserve ratios so large that the diversification effect dominates. In any case, the liquidity situation of the merged banks improves, both in terms of liquidity risk and expected needs.

The effect of the merger on the loan market depends on the relative strength of increases in market power and potential cost efficiency gains. Since the merger allows the two banks to internalise the effect of their pricing also on the demand of their companion bank, they are able to set *ceteris paribus* higher loan rates. At the same time, potential efficiency gains together with the savings of interbank financing costs through the internal money market make banks more aggressive in setting loan rates. Hence, overall loan rates may either increase or decrease depending on how strong cost reductions are. Since banks compete in strategic complements, the loan rates of the competitors move in the same direction as the loan rates of the merged banks. Regarding lending quantities, the merged banks gain market shares at the expense of competitors when loan rates fall and they lose market shares to competitors when loan rates rise. What is important to keep in mind is that consolidation changes banks' balance sheets creating (or reducing) heterogeneity, both through the reshuffling of

loan market shares and through changes in optimal reserve holdings.

The change in the size distribution of banks' balance sheets and in their balance sheet composition affects the liquidity situation in the money market. This applies to both aggregate liquidity risk (the probability of facing a liquidity shortage in the interbank market) and to expected aggregate liquidity needs (the expected amount of the liquidity needed). Mergers affect these features of aggregate banking system liquidity through two fundamental channels in the model, one working through the aggregate level of reserves and the other through the degree of asymmetry in banks' balance sheets. The *reserve channel* is directly related to individual banks' changes in liquidity management, as described above. When reserve levels are relatively low in the 'status quo' (i.e. the interbank refinancing costs are low compared to deposit financing costs), then individual banks increase reserves, the aggregate supply of liquidity goes up and both aggregate liquidity risk and average liquidity needs become more moderate. For decreasing reserves aggregate money market liquidity deteriorates.

As to the second channel, greater asymmetry or heterogeneity of bank balance sheets leads to a higher variance of aggregate liquidity demand. Whether the greater variability of liquidity demand related to this *asymmetry channel* leads to higher aggregate liquidity risk, depends again on whether the system is characterised by high or by low reserve levels. When reserves are low, then the total supply of liquidity in the money market is lower than the average liquidity demand and the greater variance of demand caused by the asymmetry effect of the merger implies less frequent liquidity shortages. In the opposite case, high refinancing costs and high reserve supply, aggregate liquidity risk is reinforced. In contrast, greater asymmetry of bank balance sheets *always* leads to higher expected liquidity needs. As this measure of money market liquidity captures the severity of shortages, the higher frequency of extreme events – in particular of very large demand realizations – ensures that the amount of liquidity that central bank will have to provide on average increases with a merger.

It turns out that the two channels can work in the same or in opposite directions. In the case of very costly interbank refinancing when mergers lead to a reduction of reserves from a relatively high level, the reserve and the asymmetry channel push towards a deterioration of aggregate liquidity in the banking system, both in terms of the frequency of shortages and in terms of their average size. Accordingly, the system becomes unambiguously more dependent on the provision of public liquidity. When, instead, for lower levels of the refinancing rate the merged banks increase reserves, the two channels work in opposite direction and the net effect on aggregate liquidity depends on their relative strength.

A main point of our paper is how these liquidity fluctuations relate to competition in the credit market. We can distinguish four possible scenarios: Bank mergers can impair both money market liquidity and loan competition, deteriorate one and improve the other or improve both. It has been argued in the empirical banking literature that particularly large mergers do not seem to be characterised by large efficiency gains. Moreover, it may be argued that actual interbank rates rarely exceed deposit rates by a wide enough margin to be in the situation where the aggregate liquidity effect is unambiguous. Therefore, in assessing the likelihood that mergers may deteriorate both banking system liquidity *and* loan competition, the relative strength of the asymmetry as compared to the reserve channel is key. What we can say is that if we face a merger wave that leads to a 'polarisation' of the banking system with very large and very small institutions, then this wave is likely to

generate such an adverse outcome. In contrast, a merger movement that leaves relatively little balance-sheet heterogeneity behind, while potentially still impairing competition in loan markets, may leave money market liquidity ‘in tact’ or even improve it.

To further explore the role of competition in the liquidity effects of bank mergers, we undertake a comparative statics exercise varying the competition parameters of the model. It turns out that, in the situation just described, a more competitive environment is favourable for aggregate banking system liquidity. As more banks or a greater substitutability of loans reduce the balance-sheet asymmetries in the system, the reserve channel is more likely to dominate and either a liquidity improving effect of a merger is strengthened or a liquidity impairing effect moderated.

As a second comparative statics exercise, we study the implications of changes in the refinancing rate (a proxy for monetary policy) for the level of loan rates. The elasticity of loan rates to the interbank rate can be interpreted as the strength of the bank lending channel in the transmission of monetary policy. The result of this experiment is that mergers *ceteris paribus* always weaken the transmission of monetary policy to loan rates. The main reason is that the internal money market makes merged banks less sensitive to the interbank refinancing costs.

The rest of the paper is structured as follows. The next section describes how our work relates to the existing literatures. Section 3 sets up the basic model. It starts with the loan competition part and finishes with the liquidity risk and interbank lending part. Section 4 presents the optimization problem and derives the equilibrium for the situation without a merger (‘status quo’). The subsequent section introduces bank mergers and studies their effects on individual institutions. Changes in reserve management, loan competition and individual liquidity risks are derived. An important part of this section is the discussion of the internal money market created by the merger. Section 6 looks at the implications of mergers for aggregate interbank market liquidity, presenting the reserve and the asymmetry channels. The comparative statics are conducted in section 7. We consider changes in the competition parameters and in the financing costs. The final section further discusses the main results and presents options for extending the model.

2 Relation to the Literature

While we are not aware of any other research work addressing the question of how the competitive changes induced by mergers affect reserve holdings and aggregate liquidity risk in the interbank money market, our paper is related to a number of issues that the literature has studied. First of all, starting with Diamond and Dybvig (1983), there is a field of research studying the role of banks as liquidity providers and the implications of this role. Recent examples are Kashyap and Stein (2002), who describe the links between banks’ liquidity provision to depositors and their liquidity provision to borrowers through credit lines; and Diamond (1997), who discusses the relationship between the activities of Diamond and Dybvig type banks and liquidity of financial markets. This literature, however, has not yet considered the implications of imperfect competition and financial consolidation for liquidity. In our model we introduce a product differentiated loan market and examine how mergers affect banks’ individual liquidity risk management (reserve holdings) and aggregate

liquidity fluctuations in the interbank market.

Second, previous literature has already studied the rationale for an interbank market and its effect on reserve holdings. For example, Bhattacharya and Gale (1987) argued that banks' liquidity shocks from depositor withdrawals are optimally coped with by a borrowing and lending program for liquid reserves. However, in the presence of both moral hazard and adverse selection the optimal contract implies under-investment in reserves compared to the first best equilibrium. Bhattacharya and Fulghieri (1994) clarify that if the timing of returns on reserves is uncertain, then the optimal second best contract can also lead to too high reserve holdings. These authors argued that the central bank has a role in healing these imperfections. In the present paper we do not re-address the incentive problems of interbank lending. We assume that at any point in time the total liquidity of the interbank market is constrained by the sum of individual reserve holdings, so that a large enough liquidity shock could lead to a shortage. For example, Allen and Gale (2000) analysed how a small unexpected regional liquidity shock can lead to a shortage of liquidity in the banking system and thus in the absence of a central bank to a contagious crisis.² Contrary to the previous interbank market literature we discuss how the likelihood and the extent of such shortages may vary with changes in market structure, as caused by bank mergers, when a central bank stands ready to compensate private market liquidity fluctuations.

Third, there is a rich industrial organization literature that analyses forms of imperfect competition and the implications of mergers for product markets (see e.g. Martin, 2001). Perry and Porter (1985) and Farrel and Shapiro (1990) show that mergers' effects on prices and quantities under imperfect competition depend on the relative strength of increases in market power and potential enhancements of firm efficiency. Interestingly, there is hardly any theoretical paper that combines these IO merger analyses with issues in banking, and - to our knowledge - there is none that relates them to liquidity. We also make the assumption that banks have market power in loan rates, using a product differentiation model as presented in Shubik (1980). Because of the usually extremely passive behaviour of retail bank depositors, we think that it is more loan competition that drives bank behaviour. In addition to the empirical evidence showing the sensitivity of loan rates to mergers and concentration,³ banks' market power in loan rates is justified through relationship lending patterns (see Sharpe, 1990, and Rajan, 1992, for theoretical justifications and Petersen and Rajan, 1994, for empirical evidence). However, we find it more realistic to think of degrees of imperfect substitutability between loans granted by different banks rather than going all the way to a monopolist bank, as often done before. Our model of bank mergers builds on Deneckere and Davidson (1985). This means that we regard it as more realistic that loan rates are the strategic variables employed by banks rather than loan quantities. We extend the Deneckere and Davidson approach to cost efficiencies in order to ensure that our results can capture both efficiency enhancing and non-efficiency enhancing mergers. The main novelty of our paper, however, is to combine such IO merger analysis with the area of banking liquidity.

²Freixas et al. (2000) discuss similar propagation risks in interbank lending schemes, covering also insolvent banks.

³For example, Edwards (1964) and Hannan (1991) find that bank concentration increases loan rates in the United States. More recently, Kahn et al. (2001) discover that US bank mergers increase interest rates for personal loans and Sapienza (2002) shows that large mergers in Italy put upward pressure on rates to be paid on credit lines.

Fourth, the expansion of firms through mergers can lead to the creation of internal capital markets. For example, Gertner, Scharfstein and Stein (1994) as well as Stein (1997) have discussed the potentially efficiency-enhancing role of such internal markets for capital.⁴ The banking literature has first related the existence of internal capital markets in large banks to lending activity. For example, Houston, James and Marcus (1997) find evidence that loan growth at subsidiaries of US bank holding companies (BHCs) is more sensitive to the holding company's cash flow than to the subsidiaries' own cash flow. Then the literature has related the existence of internal capital markets to banks' access to financing. Campiello (forthcoming) shows that the funding of loans by small affiliates of US BHCs (including at least one large bank affiliate) are less sensitive to affiliate-level cash flows than the case for independent banks of comparable size. In other words, the BHC affiliates tend to be less credit constrained than the independent banks. Focusing on short term assets only, we show with theoretical means how the creation of an internal money market can cushion external liquidity shocks and affect liquidity choices, thus aggregate liquidity risk.

Fifth, aggregate liquidity risk is related to financial stability. Although we are not covering solvency problems in our model, in practice severe liquidity problems may still cause default when there is no adequate intervention. The relationship between competition and bank stability has been studied in the 'charter value' literature, basically saying that some monopoly rents are desirable in banking to reduce incentives for excessive risk-taking (see e.g. Keeley, 1990; Hellman et al., 2000 or Matutes and Vives, 2000). The paper by Perotti and Suarez (forthcoming) argues that a succession of bank takeovers arranged by supervisors and an active entry policy can be successful in ensuring stability and keeping competition in tact. The balance of more recent empirical work indicates that less competitive banking systems are not necessarily more stable.⁵ Our model can link monopoly rents in loan competition (through concentration) to individual and aggregate liquidity fluctuations, as one factor in banking system stability.

Finally, there is an ongoing debate about the bank lending channel in the transmission of monetary policy to the economy (see e.g. Bernanke and Blinder, 1988, Bernanke and Gertler, 1995, and Stein, 1998). A contractionary monetary policy would not only work through the traditional (bond market) interest rate effect on investment, so the argument goes, but also be reinforced through the effect of an inward shift of banks' supply schedules on bank-dependent borrowers. Two ways in which the literature has tried to identify the bank lending channel is through bank size (see e.g. Kashyap and Stein, 1995, or Kishan and Opiela, 2000) and through bank liquidity (see e.g. Kashyap and Stein, 2000). For example, small banks are often assumed not to be able to switch to alternative sources of funding during a tightening of monetary policy. Similarly, banks with a smaller share of liquid assets on their balance sheet are often considered to be less able to maintain lending during a monetary tightening. Our theory allows for some comparative statics of how changes in banks' refinancing costs (a proxy for the stance of monetary policy) affects loan supply.

⁴Scharfstein and Stein (2000) as well as Rajan, Servaes and Zingales (2000) warned that internal capital markets could become inefficient, if internal incentive problems and power struggles led to excessive cross-divisional subsidies. However, more recent empirical results indicate that 'value destruction' in firms does not seem to be related to consolidation (see e.g. Graham, Lemmon and Wolf, 2002).

⁵Carletti and Hartmann (forthcoming) provide a more comprehensive review of the literature on competition and stability in banking.

More specifically, we compare the strength of this bank lending channel in the ‘status quo’ with its strength after consolidation has happened. Our general set-up has some elements in common with Chiappori et al. (1995), who argue in a spatial competition model that deposit rate regulations can lead to a reduced transmission of monetary policy to lending rates. The only other micro theoretical model of the bank lending channel we are aware of (Stein, 1998) relies on adverse selection and also does not address the effects of bank consolidation (only a monopoly bank is considered).

3 The Model

Consider a three date ($T = 0, 1, 2$) economy with three classes of risk neutral agents: N banks ($N > 3$), numerous entrepreneurs and numerous individuals. At date 0 banks raise funds from individuals and invest the proceeds in loans to entrepreneurs and liquid short term assets denoted as reserves. Thus, the balance sheet accounting identity for each bank i is

$$L_i + R_i = D_i. \quad (1)$$

L_i denotes loans, R_i liquid reserves and D_i deposits.

Competition in the loan market

Banks compete in prices in the loan market. As their loans are differentiated, banks exercise some market power. In practise, the differentiation and related imperfect substitutability may emerge from long-term lending relationships (see e.g. Sharpe, 1990, and Rajan, 1992), specialisation in certain types of lending (e.g. to small/large firms or to different sectors) or in certain geographical areas. Following a standard product differentiation approach (see Shubik, 1980), we then assume that each bank i faces a linear demand for loans given by

$$L_i = l - \gamma \left(r_i^L - \frac{1}{N} \sum_{j=1}^N r_j^L \right). \quad (2)$$

The variable L_i indicates the quantity of bank i 's loans demanded by entrepreneurs, r_i^L is the loan rate charged by bank i , r_j^L is the loan rate charged by banks j ($j = 1 \dots i \dots N$) and the parameter $\gamma \geq 0$ represents the degree of substitutability of loans for entrepreneurs. The larger (smaller) γ the more (less) substitutable the loans are. Note that expression (2) implies a constant aggregate demand of loans $\sum_{i=1}^N L_i = Nl$.⁶

Processing loans involves a per-unit provision cost c for banks, which can be thought of a set up cost or a monitoring cost. Loans mature at date 2 and are completely illiquid, as they yield nothing if liquidated before maturity. As discussed further below, this assumption excludes the occurrence of bank runs.

Deposits, individual liquidity shocks and reserve holding

⁶In this respect our modelling approach is similar to the spatial competition approach by Salop (1979), often used in the banking literature.

Banks raise deposits in N identical ‘regions’. There is a large number of potential depositors in every region, each endowed with one unit of funds at date 0. A region can not only be interpreted as a geographical area but also as a specific segment of the population or an industry sector in which a bank specialises for its deposit business. Deposits are offered demandable contracts, which pay just the initial investment in case of withdrawal at date 1 and a fixed (net) rate r^D in case of withdrawal at date 2. The rate r^D can be thought of the reservation value of depositors (the return of another investment opportunity) or alternatively the equilibrium rate in a competition game between banks and other deposit-taking financial institutions.

Following standard banking theory, we introduce liquidity shocks: A fraction δ_i of depositors at each bank/region develops a preference for early consumption and therefore withdraws at date 1.⁷ The remaining $1 - \delta_i$ depositors value consumption only at date 2 and leave their funds at the bank a period longer. To introduce uncertainty at the level of individual banks and in the aggregate, we assume δ_i to be stochastic and, specifically, to be uniformly distributed between 0 and 1 and i.i.d. across banks. All uncertainty is resolved at date 1 when the liquidity shocks materialise.⁸

In sum, each bank i faces an uncertain (*gross*) demand for liquidity at date 1 given by

$$x_i = \delta_i D_i. \quad (3)$$

Banks use their reserve holdings R_i to satisfy this demand. In theoretical terms reserves represent a simple storage technology that transfers the value of investments from one period to the next. Practically, we may think of cash, reserve holdings at the central bank or short-term government securities.⁹ Since Given the stochastic nature of δ_i , the realised demand for liquidity x_i may sometimes exceed R_i and the bank will require to fund the difference. From an ex ante perspective, we can express a bank’s *liquidity risk* – the probability that it experiences a liquidity shortage at date 1 – as

$$\phi_i = \text{prob}(x_i > R_i) = \int_{R_i}^{D_i} f(x_i) dx_i. \quad (4)$$

A bank’s *resiliency* against liquidity shocks – the probability that the bank does not experience a liquidity shortage – is then $1 - \phi_i$. Also, we can define a bank’s *expected need of refinancing* – the expected size of liquidity needed, given that a shortage occurred – as

$$\omega_i = \int_{R_i}^{D_i} (x_i - R_i) f(x_i) dx_i. \quad (5)$$

⁷As in Diamond and Dybvig (1983) δ_i may represent the stochastic fraction of the continuum of depositors that desires to consume at $T = 1$, where each agent has an equal and independent probability of withdrawing early.

⁸Alternatively, δ_i can be seen as representing a regional macro shock. For example, weather conditions may change the general consumption needs across the region, so that each depositor withdraws a fraction δ_i from his initial investment.

⁹We do not consider regulatory minimum reserve requirements because we are interested in a broader concept of liquidity. Since the interest rate on the storage technology needs not be zero for the results of the model to hold, we can also include safe and low yielding assets other than cash in the list of bank reserves.

Interbank refinancing and aggregate liquidity

Banks can meet excess liquidity demands at date 1 by borrowing in an interbank market.¹⁰ As liquidity shocks are independently distributed across banks, there is room for reshuffling liquidity from banks with excess reserves ($x_i < R_i$) to banks with reserve shortages ($x_i > R_i$). This mechanism is sufficient as long as the aggregate demand for liquidity is not larger than the aggregate supply of liquidity, the sum of individual banks' reserves in the system, that is when

$$\sum_{i=1}^N x_i \leq \sum_{i=1}^N R_i \quad (6)$$

The presence of aggregate uncertainty, however, implies that there are states of the world in which inequality (6) is violated and an aggregate shortage of liquidity occurs. We assume that in these situations the central bank provides any missing liquidity at the rate justified by the current stance of monetary policy.

For the sake of tractability, we assume the interbank market to be perfectly competitive and the central bank to target the competitive interbank rate, so that lenders break even and borrowers bear the rate r^I . From an ex ante perspective, this assumption implies that banks do not invest in reserves at date 0 to make profits, but they only hold reserves to protect themselves against shocks.¹¹

We are interested in studying the aggregate liquidity fluctuations occurring in the private banking system. Our focus is on both the frequency and the size of these liquidity fluctuations. We capture the frequency through the aggregate (or systemic) liquidity risk

$$\Phi = \text{prob} \left(\sum_{i=1}^N x_i > \sum_{i=1}^N R_i \right), \quad (7)$$

and the size or severity through the conditional expected aggregate liquidity needs

$$\Omega = E \left[\sum_{i=1}^N x_i - \sum_{i=1}^N R_i \mid \sum_{i=1}^N x_i > \sum_{i=1}^N R_i \right]. \quad (8)$$

Liquidity risk (7) and expected liquidity needs (8) can also be interpreted as measures of the degree to which the banking system depends on the public supply of liquidity.¹²

¹⁰We are thinking here of the ultra-short term interbank deposit market, like the overnight market in the euro area or the Fed funds market in the United States.

¹¹The rate r^I then reflects interbank search and loan processing costs, including transaction costs that may be charged by money market makers or brokers, banks' disutility of lending, etc. In a richer model, r^I would also include the risk premium on interbank loans. An alternative interpretation of this assumption is that the interbank market is relatively passive, in the sense that 'short' banks demand what they need and 'long' banks offer the funds they have without serious attempts to make money. This picture seems to be a fairly good description of most banks' behaviour in the euro overnight market.

¹²Holmstrom and Tirole (1998) examine another liquidity provision by public authorities. They ask whether in the presence of aggregate uncertainty government debt management should be used as a means to better meet the liquidity needs of the productive sector.

The overall timing and structure of the model is summarized in figure 1. At date 0 banks compete in prices in the loan market, choose reserve holdings and raise the necessary deposits, so that the accounting identity (1) is fulfilled.¹³ At date 1 banks are hit by liquidity shocks, borrow or lend in the interbank market and, if necessary receive additional refinancing from the central bank. At date 2 all claims (loans, remaining deposits) are settled and profits materialise. Since our main interest is in liquidity issues, we work under the assumption that loan business is sufficiently profitable to repay creditors in the retail deposit and in the wholesale interbank market throughout.¹⁴

T=0	T=1	T=2
price competition in the loan market, choice of R_i , $D_i = L_i + R_i$ are raised	shocks δ_i materialise, interbank refinancing takes place and central bank may lend	loans mature, remaining depositors withdraw and profits materialise

Figure 1: Structure and timing of the model

4 The Status Quo

We now line out the problem to be solved and characterise the (symmetric) equilibrium in the loan market and the optimal reserve holdings when all banks are identical.

We start with describing some features of the model. First, note that although banks operate over three dates, we can focus only on the $T = 0$ maximisation problem, since the date 1 and 2 feasibility (or budget) constraints are never violated, for any realisation of the liquidity shock.¹⁵ Second, bank runs never occur in this model. The illiquidity of loans together with $r^D > 0$ guarantees that depositors withdraw prematurely only if hit by liquidity shocks.

With these considerations in mind, each bank i chooses at $T = 0$ its loan rate r_i^L and liquid reserves R_i simply to maximise its total expected profits¹⁶

$$\Pi_i = (r_i^L - c)L_i - \int_{R_i}^{D_i} r^I(x_i - R_i)f(x_i)dx_i - r^D D_i(1 - E(\delta_i)), \quad (9)$$

¹³Letting reserves be chosen after competition in the loan market, or before but being not observable, leads to the same results. Letting reserves be chosen before loan market competition and be observable would lead to the standard precommitment effect where, since banks compete in strategic complements, they reduce reserves (or deposits) to strengthen capacity constraints and soften loan market competition (Vives, 1986). Since this effect is well known, we avoid precommitment considerations to focus on liquidity choices. Also, it does not seem plausible from a practical perspective that short-term reserves are observable and chosen before long-term loan decisions are made.

¹⁴It is easy to show that this simply requires $r_i^L - c \geq r^I$.

¹⁵This follows from our assumptions that the interbank market rate is small relative to the loan rate, and that the central bank completes the interbank market.

¹⁶For simplicity, the intertemporal discount factor is normalised to one.

where the first term represents the profits from the loan market, the second term is the expected cost of ex post refinancing when reserves are not enough, and the third term is the expected repayment to depositors leaving their funds until the final date. Taken together, the last two terms represent the bank's *total financing costs*.

In order to have non-negative reserve holdings, we restrict our attention to the quite plausible case in which $r^I > r^D$.¹⁷ We have the following:¹⁸

Proposition 1 *The symmetric status quo equilibrium in the loan market is characterised as follows:*

$$r_{sq}^L = \frac{l}{\gamma \left(\frac{N-1}{N} \right)} + c_{sq}, \quad L_{sq} = l, \quad \Pi_{sq} = \frac{l^2}{\gamma \left(\frac{N-1}{N} \right)}$$

where $c_{sq} = c + \sqrt{r^I r^D}$. The optimal reserve holding and deposit size are:

$$R_{sq} = \left(\sqrt{\frac{r^I}{r^D}} - 1 \right) l, \quad D_{sq} = l \sqrt{\frac{r^I}{r^D}}.$$

The results are intuitive. Banks set the loan rate to balance marginal profits from granting loans with the marginal costs of providing loans, increasing the expected refinancing needs and raising deposits. The equilibrium loan rate r_{sq}^L diverges from the total marginal cost c_{sq} via the mark up $\frac{l}{\gamma \left(\frac{N-1}{N} \right)}$, which is decreasing in the number of banks N and in the substitutability parameter γ , and increasing in the equilibrium demand for loans l . The total marginal cost include the loan provision cost c and the marginal total financing costs $\sqrt{r^I r^D}$, that is the sum of the expected cost of refinancing and of raising deposits. Bank profits are positive and increasing in both the mark up and demand for loans.

In choosing reserves, banks balance the marginal benefit from reducing refinancing needs with the marginal cost of increasing deposits. As a result, in equilibrium reserves increase with the refinancing rate r^I and demand for loans L_{sq} , while decrease with the deposit rate r^D . The same holds for deposits, since they increase with reserves.

Two further implications of proposition 1 are important for later comparison with the post merger equilibrium. First, using the balance sheet equality (1), the optimal reserve choice can be expressed in terms of optimal reserve/deposit ratio k_{sq} as

$$k_{sq} = \frac{R_{sq}}{D_{sq}} = \left(1 - \sqrt{\frac{r^D}{r^I}} \right). \quad (10)$$

Second, proposition 1 implies the following:

¹⁷In the euro area the overnight deposit rate is usually several times larger than rates paid on demandable deposits. For example, since the introduction of the euro the EONIA rate has been between 3.5 and 5 times larger than average rates paid on sight deposits.

¹⁸All formal proofs are in the appendix.

Corollary 1 *In the status quo equilibrium, each bank faces liquidity risk $\phi_{sq} = \sqrt{\frac{r^D}{r^I}}$ and expected liquidity needs $\omega_{sq} = \frac{r^D}{2r^I}D_{sq} = l\sqrt{\frac{r^D}{2r^I}}$.*

The equilibrium liquidity risk ϕ_{sq} is increasing in the deposit rate r^D and decreasing in the refinancing rate r^I and. The intuition is as follows. A higher r^D induces banks to reduce reserves and thus deposits. Lower reserves reduce protection against premature demand for liquidity while lower deposits reduces the size of such demand. Because the liquidity shocks hit only a fraction δ_i of deposits, the negative effect of lower reserves dominates, thus increasing ϕ_{sq} . A similar mechanism explain the positive dependance of ϕ_{sq} on r^I .

The expected liquidity needs depend in equilibrium on the demand for loan l as well as on r^D and r^I . A larger demand for loan increases ω_{sq} as it enlarges its demand for liquidity.

5 The Effects of a Merger on Individual Banks' Behaviour

Consider now the situation in which two banks (for example bank 1 and bank 2) propose to merge, while the other $N - 2$ banks continue to act independently.¹⁹ The behaviour of the merged banks is different from the combined behaviour of the two separate banks pre-merger both with respect to loan rates and reserve holdings. When two banks merge they internalise the effects of their choices on each other and may gain cost advantages, both in terms of lower loan provision costs and a better liquidity management through an internal money market. These forces change banks' equilibrium balance sheets with respect to loan portfolios, reserve holdings and deposits. This in turns modifies the demand and supply of liquidity in the system.

We start by analysing how a merger modifies individual banks' behaviour with respect to reserve holdings, loan market equilibrium and deposit sizes. In the next section we examine the changes in aggregate demand and supply of liquidity, focusing on the effects of the merger for aggregate liquidity risk and expected aggregate needs.

5.1 The Internal Money Market and the Choice of Reserves

The merger creates room for an internal money market where the two merged banks can exchange reserves. We assume that the two banks can pool their reserves and reshuffle them as needed without any cost. This mutual insurance mechanism changes their behaviour with respect to both the choice of reserves and competition in the loan market.

Define the total deposit base of the merged banks as $D_m = D_1 + D_2$, their joint reserves as $R_m = R_1 + R_2$ and their gross expected demand of liquidity at $T = 1$ as $x_m = \delta_1 D_1 + \delta_2 D_2$.

¹⁹We analyse only the case in which two banks merge but the analysis could be generalised to the case in which M banks merge and $N - M$ banks continue to act independently.

The combined profits of the merged banks are then given by

$$\begin{aligned} \Pi_m = & (r_1^L - \beta c)L_1 + (r_2^L - \beta c)L_2 - \int_{R_m}^{D_m} r^I(x_m - R_m)f(x_m)dx_m \\ & - r^D [D_1(1 - E(\delta_1)) + D_2(1 - E(\delta_2))]. \end{aligned} \quad (11)$$

where the first two term represent the combined profits from the loan market with $\beta \leq 1$ being potential efficiency gains in the form of reduced loan provision costs;²⁰ the third term is the combined expected cost of refinancing and the last one is the total expected repayment to depositors.

Expression (11) depends on how total deposits D_m are raised. For any given level total reserves R_m and total deposits D_m , the choice of D_1 and D_2 affects the distribution of the demand for liquidity x_m and thereby the expected cost of refinancing. We have the following:

Lemma 1 *The merged banks optimally raise an equal amount of deposits in each region, $D_1 = D_2 = \frac{D_m}{2}$.*

The presence of an internal money market with its mutual insurance effects reduces the merged banks' expected costs of refinancing for any given levels of reserves and loans. To maximise this portfolio effect and minimise refinancing costs, the two merged banks choose symmetric deposit bases. Choosing asymmetric ones would induce asymmetry in the weights of the sum of random variables characterising the liquidity demand x_m . This would increase the variance of x_m , which in turn would increase the expected costs of refinancing.

Given $D_1 = D_2$, the merged banks set the loan rates r_1^L and r_2^L and choose R_m to maximise the combined profits in (11). We start with the optimal reserve holdings. Let $k_m = \frac{R_m}{D_m}$ denote the reserve/deposit ratio for the merged banks and recall k_{sq} the one for banks in the status quo defined in (10). Then,

Proposition 2 *The merged banks choose a higher reserve/deposit ratio than in the status quo when refinancing is relatively inexpensive, and a lower one otherwise:*

$$k_m > k_{sq} \text{ if } r^I < \frac{64}{9}r^D \text{ and } k_m < k_{sq} \text{ if } r^I > \frac{64}{9}r^D,$$

where

$$k_m = \begin{cases} z(r^I, r^D) & \text{for } r^I \leq 3r^D \\ 1 - \sqrt[3]{\frac{3}{8} \frac{r^D}{r^I}} & \text{for } r^I > 3r^D \end{cases}$$

and $z(r^I, r^D)$ is the unique real solution of the equation $z^3 - \frac{3}{2}z^2 + \frac{3}{8}(1 - \frac{r^D}{r^I}) = 0$ in the interval $(0, \frac{1}{2}]$ increasing in the ratio $\frac{r^I}{r^D}$.

²⁰We consider here only the case $\beta \leq 1$ but the analysis can be easily extended to the case $\beta > 1$ when the merger produces diseconomies in loan provision.

Somewhat surprisingly, proposition 2 shows that, unless refinancing is very costly relative to raising deposits, the merged banks hold relatively *more* reserves than before they merge. Two forces drive the change in the merged banks' choice of reserves. The first one is diversification: The independence of the liquidity shocks reduces the variance of the merged banks' liquidity demand, thus reducing the need for reserves. The second force is internalisation: In choosing the reserve holdings, the merged banks take into account that a unit of reserve can now be used to cover the liquidity demand of both banks without any cost. This increases the value of holding reserves. As long as refinancing is not too costly ($r^I < \frac{64}{9}r^D$), banks have low reserves and the internalisation effect dominates. The marginal benefit of holding reserves is higher for the merged banks, since the probability that an additional unit of reserves will be needed is for them higher (because the distribution of their liquidity demand is more concentrated around the mean). On the contrary, when refinancing is very costly ($r^I > \frac{64}{9}r^D$), the diversification effect dominates. Banks hold many reserves and the probability that the merged banks will need an additional unit of reserves is so low that they choose a lower reserve/deposit ratio.

Note that the merger does not affect the optimal reserve/deposit ration of the $N - 2$ competitors (marked below with subscript c). Because they have the same cost structure as in the status quo and expected profits given by (9), they still choose their reserves according to (10) and $k_c = k_{sq}$. Of course, the merger modifies all banks' behaviour on the loan market, as we discuss in the next section.

5.2 Competition in the loan market

We start with analysing banks' cost structure. The merged banks have two cost advantages with respect to competitors: the efficiency gains in the loan provision costs and a lower financing costs due to the internal money market. Let c_c denote the total marginal cost of each competitor and c_m denote the one of the merged banks. We have the following.

Lemma 2 *The merged banks have always financial cost advantage over the competitors, so that, independently from the potential efficiency gains in the loan market, in the post-merger equilibrium it is always $c_m < c_c = c_{sq}$, where*

$$c_c = c + \sqrt{r^I r^D} = c + \frac{r^I(1 - k_c)^2 + r^D}{2(1 - k_c)}$$

and

$$c_m = \begin{cases} \beta c + \frac{r^I(3 - 6k_m + 4k_m^3) + 3r^D}{6(1 - k_m)} & \text{for } r^I \leq 3r^D \\ \beta c + \frac{4r^I(1 - k_m)^3 + 3r^D}{6(1 - k_m)} & \text{for } r^I > 3r^D. \end{cases}$$

Analogously to before, the second terms of c_c and c_m are the marginal financing costs, that is the sum of (expected) cost of refinancing and raising deposits implied by an additional unit of loans. Due to the presence of the internal money market, these costs are lower for the

merged banks and, differently from the potential efficiency gains, give them a cost advantage whose size is endogenously determined by banks' optimal choice of reserves.

The following proposition describes the post-merger equilibrium with symmetric behaviour within the 'coalition' (merger) and among competitors.

Proposition 3 *The post-merger equilibrium with $r_1^L = r_2^L = r_m^L$ and $r_i^L = r_c^L$ for $i = 3 \dots N$ is characterised as follows:*

$$\begin{aligned} r_m^L &= \left(\frac{2N-1}{N-2}\right) \frac{l}{2\gamma} + \frac{(N-1)}{2N}c_c + \frac{(N+1)}{2N}c_m, \quad r_c^L = \left(\frac{N-1}{N-2}\right) \frac{l}{\gamma} + \frac{(N-1)}{N}c_c + \frac{1}{N}c_m; \\ L_m &= \left(\frac{2N-1}{N}\right) l + \gamma \frac{(N-1)(N-2)}{N^2}(c_c - c_m), \quad L_c = \frac{(N-1)^2}{N(N-2)}l - \gamma \frac{(N-1)}{N^2}(c_c - c_m); \\ \pi_m &= \frac{[(1-2N)l + \gamma(N-1)(N-2)(c_c - c_m)]^2}{2\gamma N^3(N-2)}, \quad \pi_c = \frac{(N-1)[N(N-1)l - \gamma(N-2)(c_c - c_m)]^2}{2\gamma N^3(N-2)^2}; \end{aligned}$$

where c_m , c_c , k_m and k_c are defined in lemma 2 and proposition 2. The optimal deposit sizes are

$$D_m = \frac{1}{1 - k_m}L_m, \quad D_c = \frac{1}{1 - k_c}L_c.$$

The results are again quite intuitive. Since banks compete in strategic complements, all the post-merger equilibrium loan rates move in the same direction. The first terms in the expressions for r_m^L and r_c^L represent the mark up that banks can charge. As the merger reduces the number of banks, the markups for all banks are now higher than in the pre-merger equilibrium (see r_{sq}^L in proposition 1). Furthermore, since the merged banks internalise the effect of a change of loan rate in one market on the other loan market, they charge a higher mark up than the competitors. The last two terms in the expressions for r_m^L and r_c^L are a weighted average of the total marginal costs of the merged banks and of competitors. The effect of the merger on equilibrium loan rates depends on the relative importance of increased market power and lower total marginal costs for the merged banks. Post-merger equilibrium loan rates increase when the efficiency gains generated by the merger, via both loan provision cost efficiency and lower financial costs, are small relative to the increase in market power and decrease otherwise. The merged banks "lead" the loan rate movement in the market. Consequently, they move their loan rates by more than competitors and their total market share change accordingly. This implies $L_m < 2L_{sq} < 2L_c$ when $r_m^L > r_c^L$ and $L_m > 2L_{sq} > 2L_c$ otherwise.²¹

The optimal deposit sizes depend on both equilibrium loan market shares and optimal reserve choices. These elements can move deposit bases in the same as well as in opposite direction. For example, when $L_m > 2L_{sq} > 2L_c$ and $k_m > k_c$, then $D_m > 2D_{sq} > D_c$. On the other hand, however, when $L_m > 2L_{sq} > 2L_c$ but $k_m < k_c$ it can be $D_m < 2D_{sq} < D_c$.

²¹It can be easily verified that the merged banks always benefit from merging, since their combined profits are always higher than their joint pre-merger profits. The other banks instead are not always better off after the merger: Their profits increase when loan rates increase (competitors take a 'free-ride', since they gain even more than the banks involved in the merger), but decrease when loan rates decrease.

The next two corollaries characterise the merged bank's liquidity risk and expected need of refinancing and compare them with the pre-merger situation. Note that because they follow the same reserve optimal rule as in the status quo, competitors face the same liquidity risk and expected liquidity needs as before merging, which from corollary 1 are equal to $\phi_c = \sqrt{\frac{r^D}{r^I}}$, $\omega_c = \frac{r^D}{2r^I} D_c$.

Corollary 2 *The merged banks are always more resilient than a single bank (and hence of two banks) in the status quo: $\phi_m < \phi_{sq}$, where*

$$\phi_m = \begin{cases} 1 - 2k_m^2 & \text{for } r^I \leq 3r^D \\ 2(1 - k_m)^2 & \text{for } r^I > 3r^D. \end{cases}$$

That is, even when the merged banks choose a lower reserve/deposit ratio than status quo one (when $r^I > \frac{64}{9}r^D$), they will still keep it sufficiently high that the portfolio effect dominates and their liquidity risk is lower than in the status quo. Given that $\phi_{sq} = \phi_m$, corollary 2 also implies that the two merged banks are more resilient than before merging and more than competitors.

Corollary 3 *The expected liquidity needs of the merged banks are*

$$\omega_m = \begin{cases} \left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3\right) D_m & \text{for } r^I \leq 3r^D \\ \frac{2}{3}(1 - k_m)^3 D_m & \text{for } r^I > 3r^D. \end{cases}$$

These are lower than those of two banks in the status quo ($\omega_m < 2\omega_{sq}$) if:

1. $\frac{D_m}{D_{sq}} < 4$ for $r^I > 3r^D$;
2. $\frac{D_m}{D_{sq}} < h$ for $r^I < 3r^D$, where $h\left(\frac{r^I}{r^D}\right) \in (2, 4)$;

and they are higher otherwise.

The merger changes the two banks' expected needs for three reasons: First, it generates an internal money market, which ceteris paribus always reduces the expected liquidity needs. Second, the merger modifies the two banks' optimal reserve/deposit ratio, which ceteris paribus reduces the expected liquidity needs for $r^I < \frac{64}{9}r^D$ (when $k_m > k_{sq}$) while increasing them for $r^I > \frac{64}{9}r^D$. Third, the merger changes the two banks' deposit base and hence the size of their liquidity demand through its effects on loan market competition.

When the cost of refinancing is relatively high ($r^I > 3r^D$), the first effect dominates unless cost advantages (efficiency gains and reduced financing costs) and competition in the loan market (degree of loan differentiation γ and number of banks N and soften the internalisation effect and increase merged banks' size) are so strong that the merged banks double their deposit base relatively to two status quo banks.

When the cost of refinancing is low ($r^I < 3r^D$), all banks -merged and non merged- choose less reserves (both k_m and k_{sq} are low), and the first two effects are less pronounced. Hence it is more likely that when the two merged banks increase in deposit size, they have higher expected liquidity needs ($h < 4$).

6 The Effects of a Merger on Aggregate Liquidity

We now turn to the effects of the merger on aggregate liquidity risk and expected liquidity needs. The merger affects aggregate liquidity through two channels.

First, the merger changes the aggregate amount of reserves relative to deposits; we call this the *reserve channel*. When looking at the system, the distinction between the money market internal to the two merged banks and the interbank money market vanishes, since the total supply of liquidity is composed of all individual reserve holding. Nevertheless, the existence of the internal money market alters the liquidity supply in the system through its effects on the merged banks' choice of reserve holdings, as shown in proposition 2. The system has a higher reserve/deposit ratio in the aggregate if the merged banks choose a higher reserve/deposit ratio than the status quo banks (for $r^I < \frac{64}{9}r^D$), and vice versa if they choose a lower reserve ratio (for $r^I > \frac{64}{9}r^D$).

Second, the merger generates an asymmetry in balance sheets across banks, which affects the distribution of the aggregate liquidity demand; we call this the *asymmetry channel*. This asymmetry is caused by two effects: The change that the merger induces in the loan market outcome; and the different amount of reserves held by the merged banks and by competitors.

The working of these channels is analyzed in depth in the following two sub-sections.

6.1 The Asymmetry Channel without Internal Capital Market

To rule out the reserve channel, in this subsection we temporarily assume that the merged banks cannot make use of the internal money market. This ensures that the merger does not generate financing cost advantages, leaving banks' decision rule for the optimal amount of reserves unchanged. Thus, we focus on the asymmetric distribution of loan market shares and deposits across banks, as caused by loan competition. We analyse how this affects the aggregate liquidity risk Φ as defined in (7), and expected aggregate liquidity needs Ω , as defined in (8).

Without an internal money market, all banks continue to choose reserves according to condition (10). Together with the inelastic aggregate demand for loans, this implies that the aggregate amounts of deposits and reserves do not change with the merger. Then, while aggregate liquidity supply remains constant, aggregate liquidity demand changes due the asymmetric distribution of deposit sizes across banks generated by loan competition.²² The aggregate liquidity demand changes from $X_{sq} = \sum_{i=1}^N \delta_i D_{sq}$ in the status quo to $X_m = \delta_1 \frac{D_m}{2} + \delta_2 \frac{D_m}{2} + \sum_{i=3}^N \delta_i D_c$ after the merger. Both X_{sq} and X_m are weighted sums of uniform random variables, but they differ in their weights (the deposit bases). This implies that the distribution of the aggregate liquidity demands changes with the merger, although it remains symmetric around the same mean because aggregate deposits remain constant. We can then state the following.

Proposition 4 *Suppose that the merged banks do not exchange reserves internally. Then, a merger:*

²²The equilibrium in the loan market is almost identical to the one described in proposition 3. The only small difference is that the costs c_m are now simply equal to $\beta c + \sqrt{r^D r^I}$.

1. *Decreases aggregate liquidity risk when the cost of refinancing is relatively low and increases it otherwise: $\Phi_m < \Phi_{sq}$ if $r^I < 4r^D$ and $\Phi_m > \Phi_{sq}$ if $r^I > 4r^D$;*
2. *Always increases expected aggregate liquidity needs: $\Omega_m > \Omega_{sq}$.*

The merger generates asymmetry in the size of banks' balance sheets by causing different loan demands across banks. This implies asymmetric weights in the sum of uniformly distributed random variables characterising the aggregate liquidity demand. As already seen in lemma 1, moving from a uniformly weighted sum of random variables (in the status quo) to a heterogeneously weighted sum of random variables (after merger) increases the variance of the total sum. Figure 2 illustrates the consequences of the increase in the variance of the aggregate liquidity demands. The merger increases the likelihood of extreme events, of very low and very high demand realizations.

Consider the case in the figure where total reserves (indicated by the vertical line) are low because $r^I < 4r^D$. It is easy to see why a higher probability mass on low liquidity demand states increases the resiliency of the system: it raises the probability that liquidity demand is below supply, as illustrated by the larger size of the area $1 - \Phi_m$ as compared to the shaded area $1 - \Phi_{sq}$. The opposite happens for the case of high interbank refinancing costs ($r^I > 4r^D$) where total reserves are higher than the expected average aggregate shocks.

[Figure 2 here]

Perhaps surprisingly after having seen the effects on liquidity risk, proposition 3 also states that the merger always worsens the liquidity dependence of the banking system in terms of amount of public liquidity needed. The intuition is that the expected aggregate liquidity need Ω depends not only on the frequency of events in which aggregate liquidity demand exceeds aggregate supply (total reserves) but also on the magnitude by which aggregate liquidity demand exceeds aggregate supply in each of these cases. The merger increases the frequency of extreme events, those where aggregate liquidity demand is very low and those where it is very high. When banks do not hold reserves these increases offset each other, so that expected aggregate liquidity needs are the same before and after the merger (just equal to the average aggregate shock). When banks hold positive reserves, the extreme events in which the aggregate liquidity demand is very small drop out from the calculation of the liquidity needs because reserves cover the aggregate liquidity demand. Hence, when reserves are positive the increased frequency of extreme events with high aggregate liquidity demand induced by the merger is not outweighed any more by the increased frequency of low demand events and the expected aggregate liquidity needs grow.

6.2 The Interaction with the Reserve Channel

In this section we consider again that merged banks use their internal money market, so that they modify their optimal reserve choice with respect to the status quo. Denote as

$$K_m = \frac{R_m + \sum_{i=3}^N R_c}{D_m + (N-2)D_c} = \frac{k_m D_m + \sum_{i=3}^N k_c D_c}{D_m + (N-2)D_c} \quad (12)$$

the aggregate reserve/deposit ratio after the merger. Since competitors choose the same reserve ratio as in the status quo ($k_c = k_{sq}$), the change in K_m is solely determined by the change in the merged banks' reserve ratio. Thus, K_m increases in the range $r^I < \frac{64}{9}r^D$, where the merged banks increase their reserve/deposit ratio ($k_m > k_{sq}$), and decreases otherwise. As a first step it is useful to characterise the effects of the change in the aggregate reserve/deposit ratio alone by focusing on the special case when the merger does not generate asymmetry in deposit bases across banks.

Lemma 3 *Suppose $D_m = 2D_c$. Then, a merger decreases aggregate liquidity risk and expected liquidity needs if $r^I < \frac{64}{9}r^D$, and increases them otherwise: $\Phi_m < \Phi_{sq}$ and $\Omega_m < \Omega_{sq}$ if $r^I > \frac{64}{9}r^D$, and vice versa if $\Phi_m > \Phi_{sq}$ and $\Omega_m > \Omega_{sq}$.*

The result is quite intuitive. When the merger does not generate asymmetry in deposits, its effect on aggregate liquidity depends entirely on the reserve channel. The aggregate liquidity supply changes but the aggregate liquidity demand remains the same. Thus, a merger reduces aggregate liquidity risk and expected aggregate liquidity needs when the merged banks keep relatively more reserves than in the status quo, thereby increasing aggregate liquidity supply. The contrary happens when the aggregate liquidity supply falls as result of the lower reserve/deposit ratio of the merged banks.

We now look at how the asymmetry channel and the reserve channel interact to determine the aggregate liquidity situation. Before doing it, two points are important. First, recall that the change in the merged banks' reserve holdings induced by the internal money market generates corresponding changes in their deposit bases, thus contributing, together with loan market competition, to the final degree of balance sheet asymmetry across banks. Importantly, these two forces – reserve changes and loan competition – do not always reinforce each other. For example, when $k_m < k_{sq}$ and when there are strong loan cost efficiencies, it can be that $D_m < 2D_{sq}$ even if $L_m > 2L_{sq}$.

Second, note that – contrary to the asymmetry effect (which always increases expected aggregate liquidity needs, the severity of potential liquidity shortages) – the reserve effect has a positive influence on aggregate liquidity when $r^I < \frac{64}{9}r^D$ and a negative influence when $r^I > \frac{64}{9}r^D$. This means that the asymmetry and reserve effects reinforce each other in worsening aggregate liquidity needs when $r^I > \frac{64}{9}r^D$, while the reserve effect contrasts the negative effects of asymmetry in the region $r^I < \frac{64}{9}r^D$. Hence, we consider below the two parameter regions separately.

Proposition 5 *Suppose the cost of refinancing is high ($r^I > \frac{64}{9}r^D$). Then, a merger leads to higher aggregate liquidity risk and higher expected aggregate liquidity needs: If $r^I > \frac{64}{9}r^D$, then $\Phi_m > \Phi_{sq}$ and $\Omega_m > \Omega_{sq}$.*

As already mentioned, when the cost of refinancing is high, the asymmetry channel and the reserve channel work in the same, negative direction: The higher variance of aggregate liquidity demand together with a lower supply of liquidity due to the fall in the merged banks' reserves make the system more vulnerable to liquidity shortages and more dependent on central bank liquidity provision.

Proposition 6 *Suppose the cost of refinancing is low ($r^I < \frac{64}{9}r^D$). Then:*

1. *There exists g , with $g \in (4, \frac{64}{9})$, such that a merger leads to lower aggregate liquidity risk when $r^I < gr^D$ and to higher risk otherwise;*
2. *For any small level of asymmetry, there exist \underline{g} and \bar{g} , with $1 < \underline{g} < \bar{g} < \frac{64}{9}$, such that if a merger determines this asymmetry when $\underline{g}r^D < r^I < \bar{g}r^D$, it reduces aggregate liquidity needs.*

When the cost of refinancing is smaller, the reserve and asymmetry channels affect aggregate liquidity in opposite directions, and the net effect depends on the relative strength of the two forces. Depending on the parameter configuration one or the other of the two effects can dominate.

Given that the merger generates a higher reserve/deposit ratio when $r^I < \frac{64}{9}r^D$, the reserve channel tends to lower both the aggregate liquidity risk and expected liquidity needs, as suggested in lemma 4. The asymmetry channel, however, as stated in proposition 4, tends to reduce the aggregate liquidity risk only when $r^I < 4r^D$, while it always increases the expected aggregate liquidity needs. Intuitively then, when the two channels interact, a merger will lead to lower aggregate liquidity risk in a larger range of parameter values than the case when only the asymmetry channel is active and to higher aggregate liquidity risk in a larger range of parameter values than the case when only the reserve channel is present.

As for expected aggregate liquidity needs, when the asymmetry generated by the merger is small, the reserve channel may dominate, so that there is a range of parameters for which the merger reduces the expected aggregate liquidity needs. The larger the asymmetry in deposits, the larger the range of parameters for which the merger increases the expected aggregate liquidity needs.

7 Comparative statics

In this section we discuss more in details how mergers, loan market competition and liquidity management interact and affect the aggregate liquidity in the system.

At the individual bank level, loan market competition and liquidity management are interdependent. The loan market equilibrium affects banks' reserve holdings (in absolute terms) by determining the amount of deposits required to finance loans, and hence the size of liquidity demands at any given level of reserves. Equilibrium reserve holdings determine banks' financing costs, expected costs of refinancing and the expected repayment to depositors, and thereby influence the equilibrium in the loan market. At the aggregate level, the competition in the loan market affects the amount of asymmetry in banks' deposit bases through the post merger equilibrium loan quantities.

Competition and Liquidity

We now look at the effects of various parameters on the post merger loan market equilibrium and liquidity. We start by analysing changes in loan market conditions on equilibrium loan rates and banks' size.

Lemma 4 *An increase in efficiency gains, market size or loan substitutability increases the merged banks' balance sheets relative to competitors': $\partial(D_m/2D_c)/\partial\beta < 0$, $\partial(D_m/2D_c)/\partial N > 0$, $\partial(D_m/2D_c)/\partial\gamma > 0$.*

The more efficiencies the merger generates (the lower β), the lower the equilibrium loan rates, the larger the loan market share of the merged banks relative to competitors. This implies larger deposit bases for the merged banks, both due to their higher loan demand and to the increase in their reserve holdings. Analogously, an increase in competition due to either an increase in the number of banks N or in loan substitutability γ reduces all equilibrium loan rates, but relatively more those charged by the merged banks, increasing their relative size.

Let's now evaluate the effects on aggregate liquidity risk and expected needs. There is a long standing debate about the size of the efficiency gains produced by bank mergers. Overall it seems that these gains are rather small, if at all positive. Thus, in the remainder of this section we look at the effects of various parameters on loan market competition and on banks' aggregate liquidity when the merged banks' deposit bases shrink compared to (two) competitors; that is, for parameter configurations that satisfy $D_m < 2D_c$.

Corollary 4 *Suppose mergers reduce merged banks' deposit bases. Then, an increase in efficiency gains, market size or loan substitutability reduces liquidity risk for $r^I \in (4, \frac{64}{9})r^D$ and reduces aggregate expected liquidity needs for $r^I < \frac{64}{9}r^D$.*

In the parameter region where $D_m < 2D_c$, the increase in the merged banks's deposit base generated by stronger efficiencies (a fall in β) reduces the asymmetry across banks and tends to reduce expected aggregate liquidity needs. For $r^I < \frac{64}{9}r^D$ this effect is reinforced by a parallel increase in the aggregate reserve/deposit ratio. Analogously, the effect of stronger cost efficiencies on liquidity risk depends on the level of the refinancing rate r^I and the cost of raising deposits r^D (it is positive with low r^I/r^D rates and negative with high ones), but in the interval $r^I \in (4, \frac{64}{9})r^D$ it is unambiguously positive. Analogously, by increasing merged banks' relative size a higher substitutability of bank loans softens the asymmetry channel and increases the aggregate reserve/deposit ratio. This reduces expected aggregate liquidity needs and increases the likelihood of a fall in liquidity risk. The same happens when the number of competing banks increase.

Taken together, the considerations above suggest that competition and liquidity concerns may go "hand in hand". When $r^I < \frac{64}{9}r^D$, which appears the most realistic parameter range, mergers are less harmful in terms of systemic liquidity when they produce efficiency gains and take place in a highly competitive environment. In other parameter configurations the effect of a more competitive environment on liquidity is ambiguous, depending on relative strength of the reserve and asymmetry channels.

Financing Costs and Liquidity

We now briefly discuss the effect of a change in the refinancing cost. It can be shown (using figure 3 in the Appendix) that the increase in the reserve/deposit ratio of the merged banks is non-monotonic in r^I . It is increasing for low levels of r^I (relative to r^D) and decreasing

for higher levels, with a maximum at $r^I \simeq 1.5r^D$. Hence, as long as the refinancing cost is above $1.5r^D$ a fall in r^I greatens the positive effects on the merged banks' reserve/deposit ratio relative to the status quo. The converse happens when r^I is below $1.5r^D$. Analogous reasoning applies to changes in the cost of raising deposits r^D .

However, changes in the refinancing cost r^I also affect the competition game on the loan market, since it determines financing costs and thereby the merged banks' cost advantage. It can be shown that a fall in r^I reduces banks' financing cost but it also reduces the merged banks' cost advantage. This leads to an upward pressure in the post merger equilibrium loan rates and to a reduction in the merged banks' loan market shares.

The effects of a fall in r^I on the asymmetry of deposit bases may be ambiguous. On the one hand, for levels of r^I above $1.5r^D$, a fall in r^I enlarges the merged banks' reserve/deposit ratio relative to the one of competitors, which tends to reduce the asymmetry. On the other hand, a lower r^I increases this asymmetry by reducing the merged banks' loan market shares relative to competitors. How the change in deposit base asymmetry and in reserve/deposit ratios affects the liquidity risk and expected aggregate needs crucially depends on the parameter values of the model.

A Digression on the Impact of Bank Consolidation on the Monetary Transmission Mechanism

An additional aspect of our model is that it creates a link between the short-term money market rate r^I and bank loan rates r_i^L . We can use this feature to briefly address a controversial channel of the monetary transmission mechanism, the bank lending channel. Some literature suggested that the transmission of monetary policy to the economy may be strengthened by the fact that policy rate changes may not only affect investment via their impact on long-term bond yields, but also via their impact on bank loan quantities and lending rates. The link between r^I and r_i^L is a simple representation of this bank lending channel. Even if in the model the aggregate demand for loans is inelastic, we may expect in a richer model a negative correlation between loan rates and loan demand.

To identify the bank lending channel in our model and analyse whether and how it is affected by bank consolidation, we need first to slightly reinterpret the model. Specifically, we now abstract from the interbank lending and borrowing by assuming that all banks directly refinance from the central bank in case they need additional liquidity. Refinancing still occurs at the rate r^I , which we now consider to be set directly by the central bank at the level justified by monetary policy stances.

Let us now first identify the bank lending channel before a merger takes place. This is simply the sensitivity of the loan rate to the refinancing rate r^I in the status quo. From proposition 1, we have

$$\frac{\partial r_{sq}^L}{\partial r^I} = \frac{r^D}{2\sqrt{r^D r^I}} > 0 \quad (13)$$

As this derivative is positive, monetary policy is transmitted to bank lending in the model. However, it can easily be verified that (13) is smaller than unity, as long as $r^D < 4r^I$. Since

based on ample empirical evidence we have assumed $r^D < r^I$ throughout, monetary policy is transmitted less than perfectly to the loan market in the model.

We can now analyse how bank consolidation affects the bank lending channel. Do mergers strengthen or weaken the transmission of monetary policy to bank loan rates? Since banks compete in strategic complements on the loan market and the merged banks move their loan rates more than competitors, we can answer this question by simply looking at the effects of a change in r^I on the merged banks' loan rate r_m^L . If this rate is less sensitive to a change in r^I than loan rates in the status quo, then we can be sure that also the rates charged by competitors will be so. From proposition 3, and using proposition 2 and lemma 2, we find:

$$0 < \frac{\partial r_m^L}{\partial r^I} < \frac{\partial r_{sq}^L}{\partial r^I}. \quad (14)$$

On the basis of the previous argument, (14) tells us that financial consolidation weakens the strength of the bank lending channel. The intuition for this result follows from the creation of the internal money market. Assume the central bank wants to tighten monetary policy and increases the refinancing rate. Since the merged banks can exchange reserves internally, they are less dependent on refinancing from the central bank and therefore less affected by the increase in refinancing costs. This reduces the sensitivity of their loan rate to the refinancing rate r^I and, consequently, the sensitivity of the loan rate charged by competitors.

This exploratory analysis of the bank lending channel leads us to three conclusions. First, to the extent that this channel played an important role in the monetary transmission mechanism before banking sector consolidation picked up, this consolidation should reduce *ceteris paribus* the effectiveness of monetary policy.²³ Second, while there is some evidence in favor of the bank lending channel in the United States (Kashyap and Stein, 1995 and 2000; Kishan and Opiela, 2000), the same does not generally apply to European countries (Favero et al., 1999; Ehrmann et al., 2001). This on the surface surprising results – in the light of the greater bank orientation of the European financial system – has been explained *inter alia* by the fact that many small European banks are organized in networks, where a large central institution pools and manages the liquidity of member banks (see Ehrmann and Worms, 2001). Our comparison of the status quo with the post-merger situation (involving an internal money market), can be seen as analogous to the comparison of countries not having bank networks with countries having important bank networks (such as France or Germany). Hence, our model can provide a theoretical justification for a weak bank lending channel in European countries. Finally, the very few previous theoretical analyses of the bank lending channel are mainly based on the distributional effects of monetary policy when bank heterogeneity is caused by asymmetric information (see e.g. Stein, 1998). Our model, in contrast, shows that the differential effects of monetary policy can also be caused by imperfect competition and heterogeneity in bank sizes, irrespective of asymmetric information.

²³This does not mean of course that monetary policy generally has become less effective over the last decade. There may have happened other things in parallel, which can have the opposite effect.

8 Discussion

In this section we want to further discuss some of our modelling choices. In particular, we consider the case of mergers that reduce balance sheet asymmetry in the banking system, we ask whether changing some assumptions would affect the sizes of some of the key thresholds in the model and we address the relevance of a less efficient internal money market.

In our presentation of the model, we introduce a merger in a situation where all banks are identical *ex ante*. This means that the merger leads to some degree of asymmetry or heterogeneity in banks' sizes. In doing this we have very large mergers in mind, or even waves of large mergers. Clearly, not all mergers lead to a more asymmetric banking system. For example, in a situation where the system is composed of a group of small banks and another group of large banks mergers among the small banks could have the opposite effect. This configuration reverses the functioning of the asymmetry channel described in section 6. A merger making the banking system more symmetric is, *ceteris paribus*, more likely to moderate aggregate liquidity fluctuations. Overall, however, financial consolidation can still cause greater liquidity risk and larger average liquidity needs and thereby increase the dependence of the banking sector on public liquidity when it induces a reduction of bank reserve holdings.

We now discuss the size of the thresholds that determine whether bank reserves increase or decrease. It is shown in section 5 how the presence of an internal money market may lead to an increase in the reserve/deposit ratio of the merged banks, and thus to a larger total supply of liquidity in the system. This increase appears to dominate in the model, that is it occurs for empirically more plausible ranges of interbank and deposit rates ($r^I < \frac{64}{9}r^D$). It is important to note though that the absolute size of the threshold values are rather of an indicative nature. The reason is that the relaxation of some simplifying assumptions can change those levels. First, the exact value of the thresholds depends on the distribution of liquidity shocks. We have assumed δ_i to be uniformly distributed on the support $[0, 1]$. Limiting the support to a fraction of the unit interval, as sometimes done in the literature, would reduce the threshold level above which reserves fall with the merger. Assuming another symmetric density function would also change the cut off levels, although it would not change any qualitative results. Second, in the model we neglect price effects in the choice of reserves by considering that all banks pay the same rate r^I to obtain liquidity. It might be argued that in reality this need not always be the case. Large banks (in our case merged banks) might pay a mildly lower rate, e.g. because they may be perceived as safer. When present, this force would act against the internalisation effect of the internal money market, restricting the range of parameters where merged banks increase reserves.

In a similar spirit, we have assumed that the internal money market works efficiently. One could imagine situations in which this efficiency is reduced, say, by banks' internal agency problems. Introducing such problems could weaken the internalisation effect of the internal money market and thereby moderate the building-up of reserves. However, unless an internal money market breaks down entirely – which may be regarded as a rather unlikely event – the reserve channel will nevertheless work.

References

- [1] Allen F. and D. Gale, 2000, “Financial Contagion”, *Journal of Political Economy*, 108(1), 1-33
- [2] Bhattacharya S. and D. Gale, 1987, “Preference Shocks, Liquidity and Central Bank Policy”, in W. Barnett and K. Singleton (eds) *New Approaches to Monetary Economics*, Cambridge University Press, 69-88
- [3] Bhattacharya S. and P. Fulghieri, 1994, “Uncertain Liquidity and Interbank Contracting”, *Economics Letters*, 44, 287-294
- [4] Berger A.N., R.S. Demsetz and P.E. Strahan, 1999, “The Consolidation of the Financial Services Industry: Causes, Consequences, and Implications for the Future”, *Journal of Banking and Finance*, 23(2-4), 135-94
- [5] Bernanke B.S. and A.S. Blinder, 1988, “Credit, Money, and Aggregate Demand”, *American Economic Review Papers and Proceedings*, 78(2), 435-439
- [6] Bernanke B.S. and M. Gertler, 1995, “Inside the Black Box: The Credit Channel of Monetary Policy Transmission”, *Journal of Economic Perspectives*, 9(4), 27-48
- [7] Boyd J.H. and S.L. Graham, 1996, “Consolidation in US Banking: Implications for Efficiency and Risk”, Federal Reserve Bank of Minneapolis Working Paper, no. 572, December
- [8] Bradley D.M. and R.C. Gupta, forthcoming, “On the Distribution of the Sum of n Non-Identically Distributed Uniform Random Variables”, forthcoming in *Annals of the Institute of Statistical Mathematics*
- [9] Carletti E. and P. Hartmann, 2001, “Competition and Stability: What’s Special about Banking?”, paper presented at the Bank of England/LSE Financial Markets Group conference in honour of Charles Goodhart, London, 15-16 November, forthcoming in the conference volume (ECB Working Paper, no. 146, July 2002)
- [10] Carletti E., P. Hartmann and G. Spagnolo, 2002, “Implications of the Bank Merger Wave for Competition and Stability”, in *Risk Measurement and Systemic Risk*, Proceedings of the Third Joint Central Bank Research Conference, 38-50
- [11] Chiappori P.-A., D. Perez-Castrillo and T. Verdier, 1995, “Spatial Competition in the Banking System: Localization, Cross Subsidies and the Regulation of Deposit Rates”, *European Economic Review*, 39, 889-918
- [12] Corvoisier S. and R. Gropp, 2001, “Bank Concentration and Retail Interest Rates”, ECB Working Paper, no. 72, European Central Bank, Frankfurt, July
- [13] Cruickshank D., 1999, Review of Banking Services in the UK; Competition and Regulation: An Interim Report, Interim Report to the Chancellor of the Exchequer, The Stationary Office, London, July

- [14] De Bandt O. and P. Hartmann, 2002, "Systemic Risk in Banking: A Survey", in Goodhart C. and G. Illing (eds.), *Financial Crises, Contagion and the Lender of Last Resort: A Reader*, Oxford University Press, Oxford, 249-297
- [15] Deneckere R. and C. Davidson, 1985, "Incentives to Form Coalitions with Bertrand Competition", *Rand Journal of Economics*, 16(4), 473-486
- [16] Dermine J., 2000, "Bank Mergers in Europe, the Public Policy Issues", *Journal of Common Market Studies*, 409-425
- [17] Diamond D.W., 1997, "Liquidity, Banks, and Markets", *Journal of Political Economy*, 105(5), 928-956
- [18] Diamond D.W. and P.H. Dybvig, 1983, "Bank Runs, Deposit Insurance and Liquidity", *Journal of Political Economy*, 91(3), 401-419
- [19] Diamond D.W. and R.R. Rajan, 2002, "Liquidity Shortages and Banking Crises", NBER Working Paper, no. 8937, May
- [20] Edwards F.R., 1964, "Concentration in Banking and its Effects on Business Loans", *Review of Economics and Statistics*, 46, 294-300
- [21] European Central Bank, 2000, "Mergers and Acquisitions Involving the EU Banking Industry - Facts and Implications", Frankfurt, December
- [22] European Central Bank, 2002, "Developments in Banks' Liquidity Profile and Management", Frankfurt, May
- [23] Ehrmann M. and A. Worms, 2001, "Interbank Lending and Monetary Policy Transmission: Evidence for Germany", ECB Working Paper, no. 73, Frankfurt, July
- [24] Ehrmann M., L. Gambacorta, J. Martinez-Pages, P. Sevestre and A. Worms, 2001, "Financial Systems and the Role of Banks in Monetary Policy Transmission in the Euro Area", ECB Working Paper, no. 105, Frankfurt, December
- [25] Farrel J. and C. Shapiro, 1990, "Horizontal Mergers: An Equilibrium Analysis", *American Economic Review*, 80(1), 197-126
- [26] Favero C.A., F. Giavazzi and L. Flabbi, 1999, "The Transmission Mechanism in Europe: Evidence from Banks' Balance Sheets", NBER Working Paper, no. 7231, Cambridge (MA), July
- [27] Freixas X., B. Parigi and J.C. Rochet, 2000, "Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank", *Journal of Money, Credit, and Banking*, 32(3-2), 611-640
- [28] Gertner R.H., D.S. Scharfstein and J.C. Stein, 1994, "Internal Versus External Capital Markets", *Quarterly Journal of Economics*, 109, 1211-1230

- [29] Graham J.R., M.L. Lemmon and J. Wolf, forthcoming, “Does Corporate Diversification Destroy Value?”, forthcoming in *Journal of Finance*
- [30] Group of Ten, 2001, Report on Consolidation in the Financial Sector (“Ferguson Report”), BIS/IMF/OECD, January
- [31] Hannan T.H. (1991), “Bank Commercial Loan Markets and the Role of Market Structure: Evidence from Surveys of Commercial Loans”, *Journal of Banking and Finance*, 15, 133-149
- [32] Hanweck G.A. and B. Shull, 1999, “The Bank Merger Movement: Efficiency, Stability and Competitive Policy Concerns”, *The Antitrust Bulletin*, 44(2), 251-284
- [33] Hartmann P., M. Manna, A. Manzanares, 2001, “The Microstructure of the Euro Money Market”, *Journal of International Money and Finance*, 20(6), 895-948
- [34] Hellman T.F., K. Murdock and J. Stiglitz, 2000, “Liberalization, Moral Hazard in Banking and Prudential Regulation: Are Capital Requirements Enough?”, *American Economic Review*, 90(1), 147-165
- [35] Holmström B. and J. Tirole, 1998, “Private and Public Supply of Liquidity”, *Journal of Political Economy*, 106(1), 1-40
- [36] Houston J., C. James and D. Marcus, 1997, “Capital Market Frictions and the Role of Internal Capital Markets in Banking”, *Journal of Financial Economics*, 46(2), 135-164
- [37] Huizinga, H.P., J.H.M. Nelissen, R. Vander Venet, 2001, “Efficiency Effects of Bank Mergers and Acquisitions in Europe”, Tinbergen Institute Discussion Paper, no. 088/3, Amsterdam and Rotterdam
- [38] Kahn C., G. Pennacchi and B. Sopranzetti (2001), “Bank Consolidation and Consumer Loan Interest Rates”, Financial Institutions Center Working Paper, no. 01-14, Wharton School, March
- [39] Kashyap A.K. and J.C. Stein, 1995, “The Impact of Monetary Policy on Bank Balance Sheets”, *Carnegie-Rochester Conference Series on Public Policy*, 42, 151-195
- [40] Kashyap A.K. and J.C. Stein, 2000, “What Do a Million Observations on Banks Say About the Transmission of Monetary Policy?”, *American Economic Review*, 90(3), 407-428
- [41] Keeley, 1990, “Deposit Insurance, Risk and Market Power in Banking”, *American Economic Review*, 80, 1183-1200
- [42] Kishan R.P. and T.P. Opiela, 2000, “Bank Size, Bank Capital, and the Bank Lending Channel”, *Journal of Money, Credit, and Banking*, 32(1), 121-141
- [43] Manzanares, Andres, 2002, “On the Effects of Asymmetric Weights on the Distribution of the Sum of n Random Variables”, mimeo, European Central Bank

- [44] Martin S., 2001, *Advanced Industrial Economics* (2nd ed.), Blackwell Publishers, Oxford
- [45] Matutes C. and X. Vives, 2000, “Imperfect Competition, Risk Taking and Regulation in Banking”, *European Economic Review*, 44(1), 1-34
- [46] Organisation for Economic Co-operation and Development, 2000, “Mergers in Financial Services”, DAFFE/CLP(2000)17, Paris, September
- [47] Perotti E.C. and J. Suarez, forthcoming, “Last Bank Standing: What Do I Gain if You Fail”, forthcoming *European Economic Review*
- [48] Perry M. and R. Porter, 1985, “Oligopoly and the Incentive for Horizontal Merger”, *American Economic Review*, 75(1), 219-227
- [49] Petersen M.A. and R.G. Rajan, 1994, “The Benefits of Lending Relationships: Evidence from Small Business Data”, *Journal of Finance*, 49, 3-27
- [50] Rajan R.G., 1992, “Insiders and Outsiders: The Choice Between Informed and Arm’s Length Debt”, *Journal of Finance*, 47, 1367-1400
- [51] Rajan R., H. Servaes and L. Zingales, 2000, “The Cost of Diversity: The Diversification Discount and Inefficient Investment”, *Journal of Finance*, 55, 35-80
- [52] Salop S., 1979, “Monopolistic Competition with Outside Goods”, *Bell Journal of Economics*, 10, 141-156
- [53] Sapienza P., 2002, “The Effects of Bank Mergers on Loan Contracts”, *Journal of Finance*, 57(1), 329-367
- [54] Scharfstein D.S. and J.C. Stein, 2000, “The Dark Side of Internal Capital Markets: Divisional Rent Seeking and Inefficient Investment”, *Journal of Finance*, 55, 2537-2564
- [55] Schargrodsky E. and F. Sturzenegger, 2000, “Banking Regulation and Competition with Product Differentiation”, *Journal of Development Economics*, 63, 85-111
- [56] Sharpe S.A., 1990, “Asymmetric Information, Bank Lending, and Implicit Contracts: A Stylized Model of Customer Relationships”, *Journal of Finance*, 45, 1069-1087
- [57] Shubik M., 1980, *Market Structure and Behaviour*, Harvard University Press, Cambridge (MA)
- [58] Stein J.C., 1997, “Internal Capital Markets and the Competition for Corporate Resources”, *Journal of Finance*, 52, 111-133
- [59] Stein, J.C., 1998, “An Adverse-Selection Model of Bank Asset and Liability Management with Implications for the Transmission of Monetary Policy”, *RAND Journal of Economics*, 29(3), 466-486
- [60] Vives X., 1986, “Commitment, Flexibility and Market Outcomes”, *International Journal of Industrial Organisation*, 4, 217-229

Appendix

Proof of Proposition 1

Using Leibiniz's rule and (1), from (9) we obtain the first order conditions with respect to the choice variables r_i^L and R_i :

$$\frac{\partial \Pi_i}{\partial r_i^L} = L_i + (r_i^L - c) \frac{\partial L_i}{\partial r_i^L} - \left[\frac{r^I L_i^2 + 2L_i R_i}{2(L_i + R_i)^2} + \frac{r^D}{2} \right] \frac{\partial L_i}{\partial r_i^L} = 0, \text{ for } i = 1 \dots N, \quad (15)$$

$$\frac{\partial \Pi_i}{\partial R_i} = r^D (L_i + R_i)^2 - r^I L_i^2 = 0, \text{ for } i = 1 \dots N. \quad (16)$$

Solving (16) for R_i gives

$$R_i = \left(\sqrt{\frac{r^I}{r^D}} - 1 \right) L_i. \quad (17)$$

Solving (15) for r_i^L in a symmetric equilibrium where $r_i^L = r_{sq}^L$ for $i = 1 \dots N$ after substituting (2) and (17) gives

$$l + (r_{sq}^L - c - \sqrt{r^I r^D}) \left(-\gamma \frac{N-1}{N} \right) = 0,$$

from which r_{sq}^L and c_{sq} follow. Substituting then r_{sq}^L in (2) gives L_{sq} . Plugging r_{sq} , L_{sq} and (17) in (9), we obtain Π_{sq} .

>From L_{sq} in (17) R_{sq} follows. Substituting R_{sq} and L_{sq} in (1) gives D_{sq} .

Proof of Corollary 1

Solving (4) and (5) gives $\phi_i = 1 - \frac{R_i}{D_i}$ and $\omega_i = \frac{(R_i)^2}{2D_i} - R_i + \frac{D_i}{2}$. Substituting the expression for R_{sq} and D_{sq} , we obtain ϕ_{sq} and ω_{sq} .

Proof of Lemma 1

We proceed in steps. First, we show that the variance of the liquidity demand x_m of the merged banks is minimum when deposits are raised symmetrically in the two regions. Second, we show that the expected liquidity needs of the merged banks (and therefore their refinancing costs) are lower when deposits are symmetric..

Define the liquidity demand of the merged banks as

$$x_m = \delta_1 \alpha D_m + \delta_2 (1 - \alpha) D_m,$$

where $\alpha \in [0, 1]$ indicates the fraction of deposits that the merged banks raise in one region and $(1 - \alpha)$ the fraction they raise in the other region. Since δ_1 and δ_2 are independent, the variance of x_m is simply

$$\begin{aligned} \text{Var}(x_m) &= \alpha^2 D_m^2 \text{Var}(\delta_1) + (1 - \alpha)^2 D_m^2 \text{Var}(\delta_2) \\ &= \text{Var}(\delta_1) [\alpha^2 D_m^2 + (1 - \alpha)^2 D_m^2], \end{aligned}$$

as $Var(\delta_1) = Var(\delta_2)$. Deriving it with respect to α , we obtain

$$\frac{\partial Var(x_m)}{\partial \alpha} = 2D^2 Var(\delta_1)(2\alpha - 1) = 0,$$

which has a minimum at $\alpha = \frac{1}{2}$.

Define now the liquidity demand of the merged banks as

$$x_{ma} = \delta_1 \alpha D_m + \delta_2 (1 - \alpha) D_m, \quad (18)$$

when $\alpha \neq \frac{1}{2}$ and as

$$x_{ms} = \delta_1 \frac{D_m}{2s} + \delta_2 \frac{D_m}{2} \quad (19)$$

when $\alpha = \frac{1}{2}$ so that deposits are symmetric in the two regions. Applying the general formula in Bradley and Gupta (forthcoming) to our case, the density functions of x_{ma} and x_{ms} can be written as (assume $\alpha < \frac{1}{2}$ without loss of generality)

$$f_{ma}(x_{ma}) = \begin{cases} \frac{x_{ma}}{\alpha(1-\alpha)D_m^2} & \text{for } x_{ma} \leq \alpha D_m \\ \frac{1}{(1-\alpha)D_m} & \text{for } \alpha D_m < x_{ma} \leq (1-\alpha)D_m \\ \frac{D_m - x_{ma}}{\alpha(1-\alpha)D_m^2} & \text{for } x_{ma} > (1-\alpha)D_m. \end{cases} \quad (20)$$

$$f_{ms}(x_{ms}) = \begin{cases} \frac{4x_{ms}}{D_m^2} & \text{for } x_{ms} \leq D_m/2 \\ \frac{4(D_m - x_{ms})}{D_m^2} & \text{for } x_{ms} > D_m/2. \end{cases} \quad (21)$$

Since $\alpha < \frac{1}{2}$, $f_{ma}(x_{ma})$ is steeper than $f_{ms}(x_{ms})$ both for $x_{ma} \leq \alpha D_m$ and $x_{ma} > (1-\alpha)D_m$, so that they do not cross each other in these intervals. In the interval $\alpha D_m < x_{ma} \leq (1-\alpha)D_m$, the two functions cross only in two points. Given that they are well behaved and symmetric around the same mean $D_m/2$ with $Var(x_{ma}) > Var(x_{ms})$, for any given level of reserves R_m it is

$$\begin{aligned} F_{ma} &> F_{ms} \text{ for any } R_m < \frac{D_m}{2} \\ F_{ma} &< F_{ms} \text{ for any } R_m > \frac{D_m}{2}, \end{aligned} \quad (22)$$

where $F_{ma} = \Pr(x_{ma} < R_m)$ and $F_{ms} = \Pr(x_{ms} < R_m)$.

Denote now as ω_{ma} and ω_{ms} the expected liquidity needs of the merged banks with asymmetric deposits and symmetric deposits respectively. We have

$$\begin{aligned} \omega_{ma} - \omega_{ms} &= \int_{R_m}^{D_m} (x_{ma} - R_m) f_{ma}(x_{ma}) d(x_{ma}) - \int_{R_m}^{D_m} (x_{ms} - R_m) f_{ms}(x_{ms}) d(x_{ms}) \\ &= \int_{R_m}^{D_m} x_{ma} f_{ma}(x_{ma}) d(x_{ma}) - \int_{R_m}^{D_m} x_{ms} f_{ms}(x_{ms}) d(x_{ms}) \\ &\quad - R_m(1 - F_{ma}(R_m)) + R_m(1 - F_{ms}(R_m)). \end{aligned} \quad (23)$$

Deriving (23) with respect to R_m gives

$$\begin{aligned}\frac{d(\omega_{ma} - \omega_{ms})}{dR_m} &= -R_m f_{ma}(R_m) + R_m f_{ms}(R_m) - (1 - F_{ma}(R_m)) \\ &\quad + R_m f_{ma}(R_m) + (1 - F_{ms}(R_m)) - R_m f_{ms}(R_m) \\ &= F_{ma}(R_m) - F_{ms}(R_m).\end{aligned}$$

From (22) it follows $\frac{d(\omega_{ma} - \omega_{ms})}{dR_m} > 0$ for $R_m < \frac{D_m}{2}$ and $\frac{d(\omega_{ma} - \omega_{ms})}{dR_m} < 0$ otherwise. This, together with $\omega_{ma} - \omega_{ms} = 0$ both for $R_m = 0$ and for $R_m = D_m$ implies $\omega_{ma} - \omega_{ms} > 0$ for all $R_m \in [0, D_m]$.

Proof of Proposition 2

Define now simply as $x_m = \delta_1 \frac{D_m}{2} + \delta_2 \frac{D_m}{2}$ the demand for liquidity of the merged banks with density function given by (21). Using Leibniz's rule, the equality $D_m = R_m + L_1 + L_2$ and $k_m = \frac{R_m}{D_m}$, the maximisation of (11) with respect to R_m can be written as

$$\frac{\partial \Pi_m}{\partial R_m} = \begin{cases} \frac{8}{3}k_m^3 - 4k_m^2 + 1 = \frac{r^D}{r^I} & \text{for } k_m \leq 1/2 \\ \frac{8}{3}(1 - k_m)^3 = \frac{r^D}{r^I} & \text{for } k_m > 1/2. \end{cases} \quad (24)$$

The term on the LHS of the equalities is the marginal benefit of increasing reserves, that is the reduction in the expected need of financing induced by a marginal increase of reserves. The term on the RHS of the equalities is the ratio between the marginal cost of raising reserves r^D and the marginal cost of refinancing r^I . From (24), we obtain k_m as in the proposition. The rearranged equation $f(z) = z^3 - \frac{3}{2}z^2 + \frac{3}{8}(1 - \frac{r^D}{r^I}) = 0$ for $k_m \leq 1/2$ has a unique solution $z(r^I, r^D)$ since $f(0) > 0$, $f(1/2) < 0$ and $f'(z) < 0$.

To compare k_m with k_{sq} , we rearrange k_{sq} given in (10) as

$$(1 - k_{sq})^2 = \frac{r^D}{r^I}, \quad (25)$$

where, as before, the LHS is the marginal benefit of increasing reserves and the RHS is the ratio between the marginal cost of raising reserves r^D and the marginal cost of refinancing r^I .

Denote as $f(k_m)$ the LHS of (24) and as $f(k_{sq})$ the LHS of (25). Plotting $f(k_m)$ and $f(k_{sq})$ for k_{sq} and k_m between 0 and 1, we get figure A1.

insert figure A1

The curves $f(k_m)$ and $f(k_{sq})$ cross only once at $k_{sq} = k_m = \frac{5}{8}$. Plugging this into (24) or (25) gives $k_{sq} = k_m$ when $r^I = \frac{64}{9}r^D$, which implies the statement 2) of the proposition.

Proof of Lemma 2

Solving the integral in (9), the total financing costs for competitors are

$$c_c L_c = \frac{r^I}{2} \frac{L_c^2}{(R_c + L_c)} + \frac{r^D}{2} (R_c + L_c). \quad (26)$$

Using $\frac{R_c}{D_c} = k_c$ and $\frac{L_c}{D_c} = 1 - k_c$ in (26) and rearranging it, we obtain $c_c = \frac{r^I(1-k_c)^2 + r^D}{2(1-k_c)}$. Analogously, solving the integral in (11) and using $\frac{R_m}{D_m} = k_m$ and $\frac{L_m}{D_m} = 1 - k_m$, we obtain c_m as in the lemma. It is easy to check that when the merged banks set k_m at the level which is optimal for competitors, it is always $c_c > c_m$. A fortiori this must be true when they set k_m optimally to minimise their financial costs.

Proof of Proposition 3

The merged banks choose r_1^L and r_2^L to maximise (11) while competitors choose r_i^L to maximise (9) where the subscript i is now c . Using proposition 2 and lemma 2, $D_m = R_m + L_1 + L_2$ and $D_c = R_c + L_c$ we can write the expected profits for the merged banks and competitors when reserves are chosen optimally as

$$\Pi_m = r_1^L L_1 + r_2^L L_2 - c_m(L_1 + L_2) \quad (27)$$

$$\Pi_c = (r_1^L - c_c)L_c. \quad (28)$$

where

$$L_m = L_1 + L_2 = \left[l - \gamma \left(r_1^L - \frac{1}{N} \sum_{j=1}^N r_j^L \right) \right] + \left[L - \gamma \left(r_2^L - \frac{1}{N} \sum_{j=1}^N r_j^L \right) \right], \quad (29)$$

and L_c is given by (2). The first order conditions are then given by

$$\frac{\partial \Pi_m}{\partial r_h^L} = L_h + (r_1^L - c_m) \frac{\partial L_1}{\partial r_h^L} + (r_2^L - c_m) \frac{\partial L_2}{\partial r_h^L} = 0 \text{ for } h = 1, 2 \quad (30)$$

$$\frac{\partial \Pi_c}{\partial r_i^L} = L_c + (r_i^L - c_c) \frac{\partial L_c}{\partial r_i^L} = 0 \text{ for } i = 3 \dots N. \quad (31)$$

We look at the post-merger equilibrium where $r_1^L = r_2^L = r_m^L$ and $r_i^L = r_c^L$. Substituting (29) in (30) and (2) in (31), we obtain the best response functions as

$$r_m^L = \frac{l}{2\gamma(\frac{N-2}{N})} + \frac{c_m}{2} + \frac{r_c^L}{2}. \quad (32)$$

$$r_c^L = \frac{l}{\gamma(\frac{N+1}{N})} + \left(\frac{N-1}{N+1}\right)c_c + \frac{2}{N+1}r_m^L. \quad (33)$$

Solving (32) and (33) gives the post-merger equilibrium loan rates r_m^L and r_c^L . Substituting r_m^L and r_c^L respectively in (29) and in (2) gives the equilibrium L_m and L_c . Substituting r_m^L and L_m in (27), r_c^L and L_c in (28) we obtain the equilibrium expected profits. Analogously, we derive D_m and D_c .

Proof of Corollary 2

Using (21), we can express the liquidity risk for the merged banks as

$$\phi_m = \Pr(x_m > R_m) = \begin{cases} 1 - \int_0^{R_m} \frac{4x_m}{D_m^2} dx_m & \text{for } r^I \leq 3r^D \\ \int_{R_m}^{D_m} \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I > 3r^D. \end{cases}$$

Solving the integrals, we obtain $\phi_m = 1 - 2\frac{R_m^2}{D_m^2}$ for $r^I \leq 3r^D$ and $2 - 4\frac{R_m}{D_m} + 2\frac{R_m^2}{D_m^2}$ for $r^I > 3r^D$. Substituting $k_m = \frac{R_m}{D_m}$ implies ϕ_m as in the corollary. Using k_m from proposition 2, we can express the merged banks' resiliency as

$$1 - \phi_m = \begin{cases} 2[z(r^I, r^D)]^2 & \text{for } r^I \leq 3r^D \\ 1 - 2\left(\sqrt[3]{\frac{3}{8} \frac{r^D}{r^I}}\right)^2 & \text{for } r^I > 3r^D. \end{cases}$$

Similarly, from corollary 1 we can write a bank's individual resiliency in the status quo as $1 - \phi_{sq} = k_{sq} = 1 - \sqrt{\frac{r^D}{r^I}}$. Plotting these expressions as a function of the ratio $\frac{r^I}{r^D}$, one immediately sees that $1 - \phi_m > 1 - \phi_{sq}$ always holds. Plots are available from the authors upon request.

Proof of Corollary 3

Using (21), we can express the combined expected liquidity needs for the merged banks as

$$\omega_m = \begin{cases} \int_{R_m}^{\frac{D_m}{2}} (x_m - R_m) \frac{4x_m}{D_m^2} dx_m + \int_{\frac{D_m}{2}}^{D_m} (x_m - R_m) \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I \leq 3r^D \\ \int_{\frac{D_m}{2}}^{D_m} (x_m - R_m) \frac{4(D_m - x_m)}{D_m^2} dx_m & \text{for } r^I > 3r^D. \end{cases}$$

Solving the integrals, we obtain $\omega_m = \frac{D_m}{2} - R_m + \frac{2}{3}\frac{R_m^3}{D_m^2}$ for $r^I \leq 3r^D$ and $\frac{2}{3}\frac{(D_m - R_m)^3}{D_m^2}$ for $r^I > 3r^D$. Substituting $k_m = \frac{R_m}{D_m}$, we obtain ω_m as in the corollary. From corollary 1 and proposition 2, we obtain

$$\omega_m - 2\omega_{sq} = \begin{cases} \left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3\right) D_m - (1 - k_{sq})^2 D_{sq} & \text{for } r^I \leq 3r^D \\ \frac{r^D}{r^I} \left(\frac{D_m}{4} - D_{sq}\right) & \text{for } r^I > 3r^D. \end{cases}$$

For $r^I > 3r^D$ it is immediate to see that $\omega_m < 2\omega_{sq}$ if $\frac{D_m}{D_{sq}} < 4$. For $r^I \leq 3r^D$, $\omega_m - 2\omega_{sq}$ can be rearranged as

$$\begin{aligned} \omega_m - 2\omega_{sq} &= (1 - k_{sq})^2 D_{sq} \left[\frac{\left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3\right) D_m}{(1 - k_{sq})^2 D_{sq}} - 1 \right] \\ &= \frac{r^D}{r^I} D_{sq} \left[\frac{r^I}{r^D} \left(\frac{1}{2} - k_m + \frac{2}{3}k_m^3 \right) \frac{D_m}{D_{sq}} - 1 \right]. \end{aligned}$$

Suppose for a moment $k_m = k_{sq}$ and $D_m = 2D_{sq}$. Then the expression simplifies to $k_{sq}D_{sq}(\frac{4}{3}k_{sq} - 1)$, which is negative because $k_{sq} < 1/2$. Denote $A = (\frac{1}{2} - k_m + \frac{2}{3}k_m^3)$. Since A is decreasing in k_m and $k_m > k_{sq}$ for $r^I \leq 3r^D$, it follows $\omega_m < 2\omega_{sq}$ for any $\frac{D_m}{D_{sq}} \leq 2$. By plotting the expression $\frac{r^I}{r^D}A\frac{D_m}{D_{sq}} - 1$ for $\frac{D_m}{D_{sq}} > 2$ and $\frac{r^I}{r^D} \in (1, 3]$, one sees that it exists a level of $\frac{D_m}{D_{sq}}$, which we denote as h , such that $\omega_m \begin{cases} \leq & \geq \end{cases} 2\omega_{sq}$ if $\frac{D_m}{D_{sq}} \begin{cases} \leq & \geq \end{cases} h$. The threshold $h \in (2, 4)$ and is increasing in $\frac{r^I}{r^D}$. The plot is available from the authors upon request.

Proof of Proposition 4

This proof is a generalisation of that of lemma 1. Recall $ND_{sq} = D_m + (N-2)D_c$ and denote it simply as D_{tot} . Also, recall $NR_{sq} = R_m + (N-2)R_c$ and denote total reserves simply as R_{tot} . Applying the formula for the distribution of a weighted sum of uniformly distributed random variables in Bradley and Gupta (forthcoming) to our model we obtain the density function of the aggregate liquidity demand in the status quo $f_{sq}(X_{sq})$ and after the merger $f_m(X_m)$ as

$$f_{sq}(X_{sq}) = \frac{1}{(N-1)!(D_{sq})^N} \sum_{i=0}^N \left[(-1)^i \binom{N}{i} (X_{sq} - iD_{sq})_+^{N-1} \right]$$

$$f_m(X_m) = \frac{\sum_{i=1}^{N-2} \left[(-1)^i \binom{N-2}{i-1} (X_m - D_m - (i-1)D_c)_+^{N-2} + \binom{N-2}{i} (X_m - iD_c)_+^{N-2} \right]}{(N-2)!D_m(D_c)^{N-2}}.$$

The two density functions are plotted in figure 2. The density $f_{sq}(X_{sq})$ is more concentrated around the mean than $f_m(X_m)$. To verify that this is always the case, we compare the variances of X_{sq} and X_m which are given by

$$Var(X_{sq}) = \sum_{i=1}^N D_{sq}^2 Var(\delta_i)$$

$$Var(X_m) = \frac{D_m^2}{4} Var(\delta_1) + \frac{D_m^2}{4} Var(\delta_m) + \sum_{i=3}^N D_c^2 Var(\delta_i)$$

$$= Var(\delta_i) \left[\frac{D_m^2}{2} + \sum_{i=3}^N D_c^2 \right],$$

since $Var(\delta_1) = Var(\delta_2) = Var(\delta_i)$. Since $D_m + \sum_{i=3}^N D_c = \sum_{i=1}^N D_{sq}$, by Lagrangian maximisation one immediately obtains $\left[\sum_{i=1}^2 \frac{D_m^2}{4} + \sum_{i=3}^N D_c^2 \right] > \sum_{i=1}^N D_{sq}^2$. Hence, it is always $Var(X_m) > Var(X_{sq})$. Since $f(X_{sq})$ and $f(X_m)$ are well behaved, in the sense that they approach a normal distribution, they intersect only in two points. This, together with the symmetry of the two density functions around the same mean $E[X_m] = E[X_{sq}] = \frac{D_{tot}}{2}$ and $Var(X_m) > Var(X_{sq})$, implies

$$\Phi_{sq} = \Pr(X_{sq} > R_{tot}) > \Phi_m = \Pr(X_m > R_{tot}) \text{ for any } R_{tot} < \frac{D_{tot}}{2},$$

and vice versa for $R_{tot} > \frac{D_{tot}}{2}$.²⁴ The first statement of proposition 4 follows immediately. Using the definition in (8), we have

$$\begin{aligned}\Omega_m - \Omega_{sq} &= \int_{R_{tot}}^{D_{tot}} (X_m - R_{tot})f_m(X_m)d(X_m) - \int_{R_{tot}}^{D_{tot}} (X_{sq} - R_{tot})f_{sq}(X_{sq})d(X_{sq}) \\ &= \int_{R_{tot}}^{D_{tot}} X_m f_m(X_m)d(X_m) - \int_{R_{tot}}^{D_{tot}} X_{sq} f_{sq}(X_{sq})d(X_{sq}) \\ &\quad - R_{tot}(1 - F_m(R_{tot})) + R_{tot}(1 - F_{sq}(R_{tot})).\end{aligned}$$

Deriving it with respect to R_{tot} gives

$$\begin{aligned}\frac{d(\Omega_m - \Omega_{sq})}{dR_{tot}} &= -R_{tot}f_m(R_{tot}) + R_{tot}f_{sq}(R_{tot}) - (1 - F_m(R_{tot})) \\ &\quad + R_{tot}f_m(R_{tot}) + (1 - F_{sq}(R_{tot})) - R_{tot}f_{sq}(R_{tot}) \\ &= F_m(R_{tot}) - F_{sq}(R_{tot}).\end{aligned}$$

As showed earlier, $F_m(R_{tot}) - F_{sq}(R_{tot}) > 0$ for $R_{tot} < \frac{D_{tot}}{2}$ and $F_m(R_{tot}) - F_{sq}(R_{tot}) < 0$ for $R_{tot} > \frac{D_{tot}}{2}$. Also, $F_m(0) = F_{sq}(0) = 0$ and $F_m(D_{tot}) = F_{sq}(D_{tot}) = 0$. This implies $\Omega_m - \Omega_{sq} > 0$ for all $R_{tot} \in [0, D_{tot}]$.

Proof of Lemma 3

Suppose first $r^I < \frac{64}{9}r^D$ and note that the aggregate reserve/deposit ratio in the status quo (which coincides with the individual banks' deposit ratio) is smaller than the one after merger. Formally,

$$k_{sq} = \frac{R_{sq}}{D_{sq}} = \frac{\sum_{i=1}^N R_{sq}}{ND_{sq}} < K_m$$

since $k_m > k_c = k_{sq}$ in this range. Consider now at the aggregate liquidity risk. When $D_m = 2D_c$, this is given by

$$\Phi_{sq} = prob\left(\sum_{i=1}^N \delta_i D_{sq} > \sum_{i=1}^N R_{sq}\right) = prob(X' < k_{sq}).$$

in the status quo and by

$$\Phi_m = prob\left(\sum_{i=1}^N \delta_i D_c > R_m + \sum_{i=3}^N R_c\right) = prob(X' < K_m),$$

after merger, where $X' = \sum_{i=1}^N \frac{\delta_i}{N}$. Since $K_m > k_{sq}$, it follows $\Phi_m < \Phi_{sq}$.

We can then express the expected aggregate liquidity needs in the status quo as

$$\Omega_{sq} = \int_{k_{sq}ND_{sq}}^{ND_{sq}} (X_{sq} - k_{sq}ND_{sq})f(X_{sq})d(X_{sq}) = ND_{sq} \int_{k_{sq}}^1 (X' - k_{sq})f(X')d(X')$$

²⁴For an alternative proof of this point, see Manzanares (2002).

Applying the same logic, the post-merger expected aggregate liquidity needs are

$$\begin{aligned}\Omega_m &= ND_c \int_{K_m}^1 (X' - K_m) f(X') d(X') \\ &= ND_{sq} (1 + (K_m - k_{sq})) \int_{K_m}^1 (X' - K_m) f(X') d(X'),\end{aligned}$$

where we have used $D_m = 2D_c$ and $D_m + (N - 2)D_c = ND_c = ND_{sq} + (K_m - k_{sq})ND_{sq}$. Given $K_m > k_{sq}$, we can write the expected aggregate liquidity needs as

$$\begin{aligned}\Omega_{sq} &= ND_{sq} \left[\int_{K_m}^1 (X' - k_{sq}) f(X') d(X') + \int_{k_{sq}}^{K_m} (X' - k_{sq}) f(X') d(X') \right] \\ &= ND_{sq} \left[\int_{K_m}^1 (X' - K_m) f(X') d(X') + (K_m - k_{sq}) \int_{K_m}^1 f(X') d(X') + \int_{k_{sq}}^{K_m} (X' - K_m) f(X') d(X'), \right]\end{aligned}$$

and, after rearranging and simplifying, we have

$$\Omega_m - \Omega_{sq} = ND_{sq} \left[\begin{aligned} &(K_m - k_{sq}) \int_{K_m}^1 (X' - K_m - 1) f(X') d(X') \\ &- \int_{k_{sq}}^{K_m} (X' - K_m) f(X') d(X') \end{aligned} \right] < 0$$

because $(X' - K_m - 1) < 0$.

Analogous steps can be followed for the case $r^I > \frac{64}{9}r^D$.

Proof of Proposition 5

Proposition 4 implies that if $k_m = k_{sq}$, then $\Phi_m > \Phi_{sq}$ and $\Omega_m > \Omega_{sq}$ for any $r^I > \frac{64}{9}r^D$. A fortiori this holds in equilibrium where $k_m < k_{sq}$, since Φ_m and Ω_m are decreasing in K_m –as defined in (12)– that falls with k_m .

Proof of Proposition 6

Statement 1. By proposition 4, $K_m = k_{sq}$ implies $\Phi_m = \Phi_{sq}$ when $r^I = 4r^D$, and $\Phi_m < \Phi_{sq}$ when $r^I < 4r^D$. Since in the range $r^I < \frac{64}{9}r^D$ $K_m > k_{sq}$, it is $\Phi_m < \Phi_{sq}$ when $r^I = 4r^D$. The strict inequality and continuity imply that there must exist a neighborhood where $r^I > 4r^D$ and $\Phi_m < \Phi_{sq}$. For $r^I > \frac{64}{9}r^D$ $\Phi_m > \Phi_{sq}$ (from proposition 5), hence there must exist $g \in (4, \frac{64}{9})$ as defined in the statement.

Statement 2. From proposition 2, $k_m = k_{sq}$ for $r^I = r^D$ and $r^I = \frac{64}{9}r^D$, and $k_m > k_{sq}$ for $r^D < r^I < \frac{64}{9}r^D$. This induces the same relation between K_m and k_{sq} , so that $K_m - k_{sq}$ is first increasing and then decreasing in the interval $r^I \in (r^D, \frac{64}{9}r^D)$. By proposition 4, when $D_m \neq 2D_c$ there is a neighborhood of $r^I = r^D$ where $\Omega_m - \Omega_{sq} > 0$. Also, when $r^I = \frac{64}{9}r^D$ and $D_m \neq 2D_c$, $\Omega_m > \Omega_{sq}$. When $r^I = r^D$ it is always $\Omega_m = \Omega_{sq} = \frac{D_{tot}}{2}$. From lemma 3, when $D_m = 2D_c$ it is $\Omega_m - \Omega_{sq} < 0$ for any $r^I \in (r^D, \frac{64}{9}r^D)$ and $\Omega_m = \Omega_{sq}$ when $r^I = \frac{64}{9}r^D$. By continuity, if one fixes a sufficiently small level of asymmetry in the deposit bases across banks ($D_m - 2D_c$ sufficiently small), then $\Omega_m - \Omega_{sq} > 0$ in an immediate neighborhood of

$r^I = r^D$. Given that $K_m - k_{sq}$ is increasing around $r^I = r^D$, there will be a higher ratio $\frac{r^I}{r^D}$, named \underline{g} in the statement, such that if the merger generates that asymmetry when $r^I = \underline{g}r^D$ then $\Omega_m - \Omega_{sq} = 0$ and $\Omega_m - \Omega_{sq} < 0$ in the immediate right neighborhood.. Again by continuity, $\Omega_m - \Omega_{sq} > 0$ in an immediate neighborhood of $r^I = \frac{64}{9}r^D$. Given that $K_m - k_{sq}$ is decreasing around $r^I = \frac{64}{9}r^D$, there will be a smaller ratio $\frac{r^I}{r^D}$, named \bar{g} , such that when $r^I = \bar{g}r^D$ then $\Omega_m - \Omega_{sq} = 0$ and $\Omega_m - \Omega_{sq} < 0$ in the immediate left neighborhood.. The statement follows.

Proof of Lemma 4

Consider the parameter β : From proposition 3, it is easy to check $\partial r_m^L / \partial \beta > 0$ and $\partial r_c^L / \partial \beta > 0$. Since banks compete in strategic complements, it is also $\partial r_m^L / \partial \beta > \partial r_c^L / \partial \beta$ and consequently $\partial L_m / \partial \beta > \partial L_c / \partial \beta$. Given $D_m = \frac{1}{1-k_m}L_m$ and $D_c = \frac{1}{1-k_c}L_c$, it follows $\partial(D_m/2D_c) / \partial \beta < 0$. Analogous reasoning applies for the parameters γ and N .

Proof of Corollary 6

When $r^I < \frac{64}{9}r^D$, the aggregate reserve/deposit ratio K_m increases (reserve channel). By lemma 3, this implies lower aggregate liquidity risk and expected needs. By lemma 4, a decrease in β (an increase in N or γ) increase the ratio $D_m/2D_c$, which, in the range $D_m < 2D_c$, reduces the asymmetry in the deposit bases and consequently the variance of the aggregate liquidity demand (asymmetry channel). This last effect tends to reduce aggregate liquidity risk for $r^I > 4r^D$ and aggregate expected liquidity needs for any $r^I < \frac{64}{9}r^D$. The statement follows.

Figure 2: Aggregate liquidity risk before and after merger

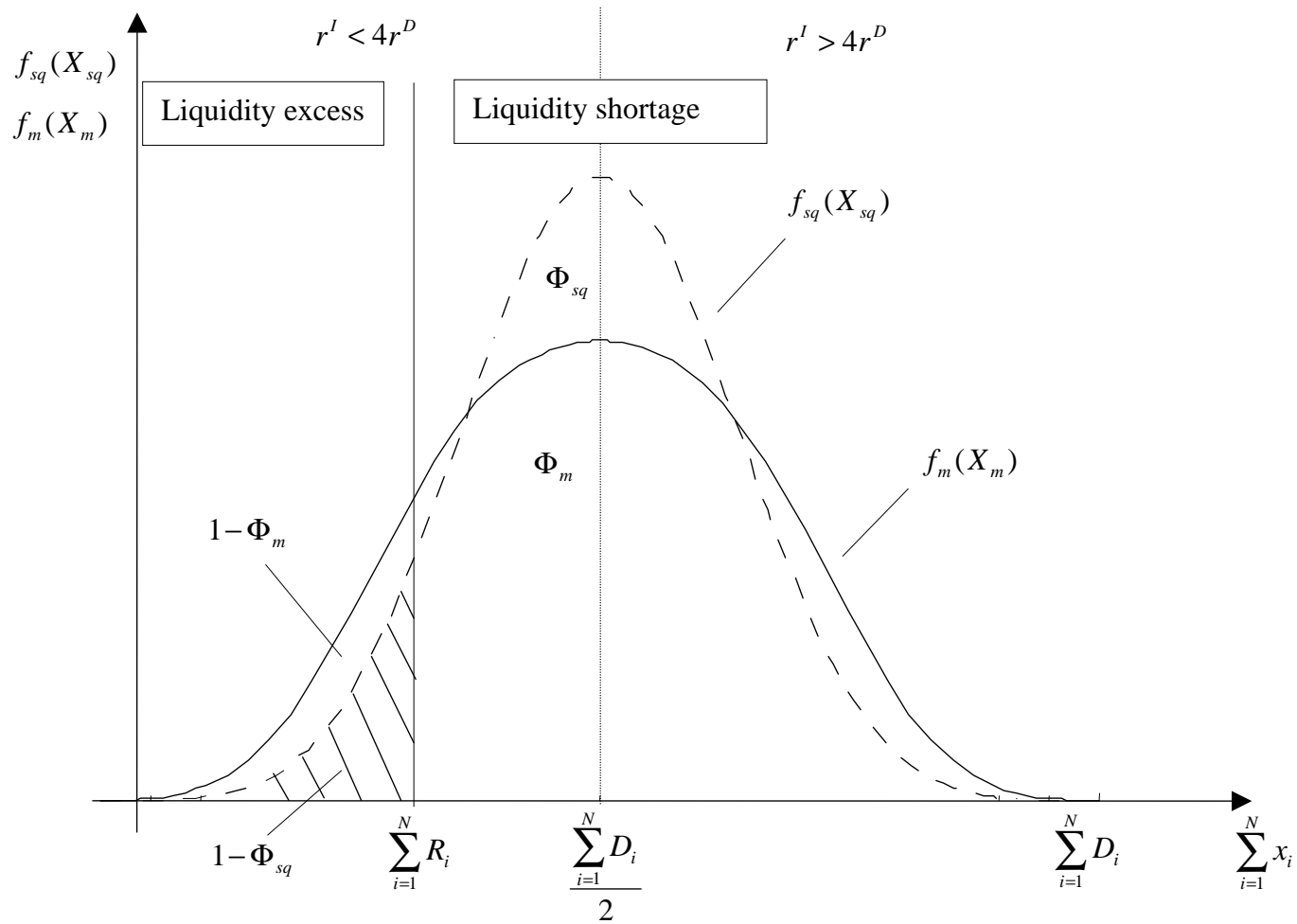


Figure 3: $f(k_{sq})$ and $f(k_{im})$

