Debt, Outside Equity, and Capital Structure under Costly State Verification*

Hans K. Hvide and Tore Leite†

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Abstract

We show that with a reformulation of the classic CSV model of financial contracting from Townsend (1979) and Gale & Hellwig (1985), it can tackle criticisms raised against it, such as lack of subgame-perfectness at the repayment stage and its inability to encompass equity contracts. Furthermore, the implications drawn are shown to be consistent with empirical regularities, such as strategic defaults of debt obligations, firms being financed by a mix of debt and equity, violations of absolute priority rules, and a low debt ratio for high risk projects. These results suggest that the CSV approach to financial contracting may have more merit to it than conceived by some earlier research.

Keywords: Capital Structure, Costly State Verification, Debt, Outside Equity, Financial Contracting.

1 Introduction

Financial contracts are typically incomplete, in that repayment to investors cannot be specified as a detailed function of all payoff relevant variables. For example, debt con-

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†Both authors: Department of Finance, Norwegian School of Economics and Business, Helleveien 30, 5045 Bergen, Norway. Email: hans.hvide@nhh.no, tore.leite@nhh.no.

tracts normally do not specify repayments as a detailed function of the financial state of the firm, but rather puts some easily describable liability on the firm's cash flow through a fixed repayment obligation. One focal approach in the literature that attempts to model such problems of incomplete contracts is the Costly State Verification (CSV) approach. The core of this approach is that, upon the date of repayment, inside investors have superior information to the outside investors about the profitability of the firm, and therefore may try to divert cash from outside investors. Of course, this may in turn create an ex-ante governance problem in that external investors may be reluctant to finance the firm. The weapon outside investors can use to mitigate the cash diversion problem is to partially or fully verify the true profitability of the firm, by e.g., demanding an audit, declaring bankruptcy, or even discharge management and take control of the operations of the firm. Such a leveling of information can only take place at a certain cost of verification. Celebrated papers by Townsend (1979) and Gale & Hellwig (1985) derives debt contracts as optimal contract under such circumstances, i.e., contracts which promises a fixed repayment, and where the creditor verifies whenever the offered repayment falls below the promised repayment.

In spite of its elegance, the classroom CSV model suffers from several shortcomings. First, as pointed out by Hart (1995), the debt contract derived under CSV relies on a commitment on the part of the lender to verify whenever the debt is not repaid in full, even if accepting a concession would be better for the lender, since verification is costly. As such, the equilibrium supporting the 'optimal contract' may involve non-Nash strategies to be played by the creditor in default states, and – perhaps equally importantly – implies that the model cannot accommodate strategic defaults of debt obligations by the borrower. Second, as also pointed out by Hart (1995), while in practice debt typically coexist with equity as a financial claim on the firm, the standard CSV model is unable to explain the use of outside equity, and hence unable to account for capital structures with both debt and outside equity on the balance sheet.[1]

The purpose of the present paper is to recast the CSV model in response to the criti-

[1] Indeed, as noted by Townsend (1979), "the [CSV] model as it stands may contribute to our understanding of closely held firms, but cannot explain the coexistence of publicly held shares and debt."

cisms above, where the two important alterations compared to Townsend (1979) and Gale & Hellwig (1985) is to require subgame perfectness and allow for stochastic monitoring at the repayment stage. We show that with these two requirements, there exists an essentially unique equilibrium where the manager offers the lender a debt repayment that depends on the true cash-flow of the firm, and the lender monitors with a probability that is increasing in the magnitude of the default. This lenience on part of the lender implies that there can be strategic defaults of debt repayments in equilibrium, defaults that sometimes pay off in that the borrower gets a fraction of the cash-flow after the creditor made a concession.

We also introduce outside equity in the CSV setting. While debt involves a fixed payment being promised to the outside investor, equity is issued with a promise to the investor of a fixed fraction of firm's cash flow. This fractional cash flow right is in turn supported by an unconditional right for the investor to intervene and verify. In the resulting equilibrium, the payout proposed to the outside investor by the entrepreneur is increasing in the true cash flow, and the outside investor monitors with a probability that is decreasing in the size of the proposed payout.

Combining debt and equity in our model allows us to consider the possibility of a joint debt and equity financing (where debt is the senior claimant), and to thereby pin down the optimal capital structure. We show that the optimal capital structure can consist of a mixture of debt and equity. Moreover, we show that the model is consistent with the optimal debt-equity ratio decreasing in the riskiness of the firm.

When a financing mix is optimal, violations of absolute priority rules can occur in equilibrium (for low realizations of the cash flow) in that outside equity receives a positive repayment even if creditors are not repayed in full. It may be noted here that the literature generating AP-violations deals with AP-violations vis a vis the inside owner-entrepreneur. In our setting, there are AP-violations in the sense that both inside and outside equity receive positive payments even though debt is not paid in full.

Other papers have taken alternative routes to solve the dilemma posed by the lack of subgame perfectness of the basic CSV contracts.[2] For example, Krasa & Villamil (2000)

[2]Gale and Hellwig (1989) impose subgame perfection in a signaling game where the cash flow is fully revealed through the repayment offer from the inside investor to the outside investor. However, Gale &

restrict the strategy space of the borrower to offering the lender either the full repayment or a zero repayment, deriving debt as the optimal contract under no commitment on the part of the lender.[3] This restriction simplifies the inference problem of the lender to the point of ensuring that he will want to verify whenever the borrower defaults on the debt contract (i.e. offers a zero repayment) so long as the verification cost that the lender must pay is not too high. In contrast, we place no apriori restrictions on the strategy space ('reports') of the borrower, but rule out the possibility of the lenders offering non-linear contracts, which is consistent with debt and equity as observed in practice.[4]

There is a literature on strategic defaults and AP-violations that will be further commented upon in the text. Others who consider outside equity and debt financing under incomplete contracting includes Fluck (1998), Myers (2000), and Anderson & Nyborg (2001), which operate in a symmetric-unverifiable information setup á la Grossman & Hart (1986) and Hart & Moore (1989). However, these papers focus on dynamic issues of repayment and do not derive an optimal mix of debt and outside equity.[5]

The rest of the paper is organized as follows. In Section 2 the basic model is presented. In Section 3 pure debt financing is considered, while in Section 4 we consider pure equity financing. Then, in Section 5 we examine a mix of debt and equity. Concluding remarks are given in Section 6.

Hellwig (1989) consider a setting where contracting plays no explicit role, in contrast we allow for (debt or equity) contracts to be written on payoffs in the verification state. Povel and Raith (2002) examine a setting where a firm's cash flow is unobservable to the creditor, and intervention by the investor has no cost to him, but leads to a loss in future benefits to manager. In equilibrium, the manager fully reveals the true cash flow to the investor, and the investor intervenes with a probability that is decreasing in the size of repayment to the investor. As with Gale & Hellwig (1989), their setting is different because verification state payoffs cannot be contracted upon.

[3]Krasa & Villamil (2000) show that debt is optimal under no commitment, while random verification is optimal under full commitment. For sake of comparison, Townsend (1979) and Gale & Hellwig (1984) consider the case with full commitment, while we consider the case with no commitment.

[4]Although contracts are linear, payouts will not be a linear function of the cash flow, due to defaults.

[5]Boyd & Smith (1999) show that the optimal contract in a CSV type of setting can involve a mix of debt and equity. However, the payoff to outside equity in their paper is only supported by the observable part of the firm's cash flow, and hence their paper cannot explain the use of equity financing to projects that generate unobservable cash-flows.

2 The basic setup

There are two stages, the investment stage and the payoff stage. Let the cash flow x in the payoff stage be a stochastic variable with density function $f(x)$, $x \in [x_L, x_H]$, where $0 < x_L < x_H$. The expected cash flow $\int_{x_L}^{x_H} xf(x)dx$ is denoted by Ex, the required investment amount is given by I, and the NPV of the project (gross of verification costs) is hence $Ex - I$. The riskless interest rate is zero, and all agents are assumed to be risk neutral. Contracts can only specify payouts to the investors in the verification state. A *(pure) debt contract* specifies the payout to the investor as $\min[D, x]$, where D is the contractible variable. The entrepreneur-manager gets the residual after the payout. A *(pure) equity contract* specifies the payout to the investor as βx, where $\beta \in (0, 1]$ is the contractible variable (and where the entrepeneur gets the residual). In other words, we model equity as a linear contract. On a basic level, linearity is consistent with what is observed in the market: the number of shares held by each investor is independent of how well the firm is doing. It is also consistent with laws protecting minority share holders, in that a small ownership share should give proportionally the same payout (e.g., dividends, liquidation, or a takeover) as a large ownership share.[6] The entrepreneur operates in a competitive market for financing, and will have a choice between debt financing and equity financing. In Section 5, we consider the case where the entrepreneur may finance the project through a mix of debt and equity.

The realized cash flow is observed freely by the manager, but can be observed by the outside investors only at a positive cost, denoted by c_D for debt, and c_E for equity. One interpretation is that c_D is a bankruptcy cost, and that c_E is the cost for investors for discharging the manager and take control of the firm. Less dramatically, c_D and c_E could reflect the creditors' and the outside investors' respective cost for performing a thorough audit. For several reasons, it is difficult to put any tight restrictions on the relative magnitude of c_D and c_E, one complication being that debt and equity holders may have different information about the operations of the firm.[7] At this point, we therefore merely

[6]The presence of executive options, which presumably are exercised when the firm is doing well, may generate a negative relation between β and x. This feature may be included in our setting, but at a significant cost of complexity and without changing any of the main insights.

[7]See for example Habib and Johnsen (2000) for a formal model in which equity specializes on gathering

assume that $0 < c_D, c_E < x_L$, i.e., that there are enough assets in the firm for the investors to cover the verification cost.[8]

For clarity of exposition, we first consider pure debt financing in Section 3, then consider pure equity financing in Section 4, and finally consider the possibility of a mixture between debt and equity in Section 5.

3 Debt

Debt is issued with a face value $D \in \Re_{++}$, along with a right on the part of the lender to verify (intervene) if D is not paid in full. We assume that the creditor will be reimbursed for the costs of collecting the contracted payment D,[9] with D representing the maximum amount that the creditor can collect net of verification costs. Thus, while the contract specifies a payoff $\min[D, x]$, the creditor obtains $\min[D + c_D, x] - c_D = \min[D, x - c_D]$.

First the parties agree upon a debt obligation D (taken as given at this point). Then the true cash flow is realized and observed privately by the entrepreneur. The entrepreneur makes a repayment offer $\tilde{D} : [x_L, x_H] \rightarrow [0, x_H]$. We impose limited liability on the entrepeneur, so that $\tilde{D} \leq x$. Notice that the entrepeneur making a repayment offer $\tilde{D} < D$ is equivalent to proposing for the creditor to make a concession $D - \tilde{D}$ on the debt claim. Given an offer $\tilde{D} < D$ by the entrepreneur, the creditor either accepts or rejects the concession proposal. If the creditor accepts, he receives $\tilde{D}$, and the manager gets the residual $x - \tilde{D}$. If the creditor rejects, he verifies and receives a payoff according to the written contract.[10] A strategy for the creditor is an accept probability $Q(\tilde{D})$, where $Q(.)$

information about the firm in its primary use and debt on its alternative use (which may include the firm's liquidation value), an argument suggesting that c_E is lower than c_D.

Another reason for c_E being different from c_D is that since the control rights for debt and equity differ, creditors and equity holders may have different incentives to invest in a cheap monitoring technology ex-ante.

[8] The restriction $c_D, c_E < x_L$ could be made endogenous by requring the entrepeneur to borrow more than I, in order to keep a liquidity reserve for bad states.

[9] This feature is consistent with the bankruptcy law in most countries, but at any extent our results would be the same if the creditor pays the verification cost.

[10] Potentially, there is a third action open to the creditor, namely to put a counter-offer on the table (after which the manager could accept or propose a counter-counter offer, and so on). By neglecting the possibility of such counter-offers, we are implicitly assuming that the cost of making such counter-offers are significant (or, alternatively, they are significant compared to the cost for the manager). Our approach here is similar to that in Anderson & Sundaresan (1996) and Mella-Barral & Perraudin (1997).

is a mapping from the set of possible repayments $[0, x_H]$ to a probability on $[0, 1]$. For $\tilde{D} \geq D$, the contract dictates that $Q(\tilde{D}) = 1$. For $\tilde{D} < D$, then $Q(\tilde{D})$ is the probability that the creditor accepts the concession on the debt claim proposed by the manager.

We rule out pre-commitment in the verification strategy $Q(.)$ by considering subgame-perfect equilibria that involves Nash play in all reachable subgames (each possible offer by the manager is the starting node of a different subgame).[11] Such subgame perfect equilibria must involve stochastic monitoring by the creditor for offers below D.[12] Consequently, for $\tilde{D}(x; D)$, for brevity written $\tilde{D}(x)$, to be part of an equilibrium, the creditor must be indifferent between accepting and rejecting the offer, and the only candidate strategy for the creditor in a subgame perfect debt equilibrium is

$$\tilde{D}(x) = \begin{cases} x - c_D \text{ for } x \in [x_L, D + c_D] \\ D, \text{ for } x \in [D + c_D, x_H] \end{cases} \tag{1}$$

Since the function $\tilde{D}(x)$ is strictly increasing (for the lower interval of x), an offer implicitly defines a reported cash flow, $\hat{x}$.

Applied to this setting, the Revelation Principle implies that we can restrict attention to 'direct' mechanisms where the borrower truthfully reports the cash flow to the creditor, and where the distribution of cash is a function of the report by the entrepreneur. Consequently, the question is now whether there exists a function $Q(.)$ such that the manager has incentives to play the strategy in (1). It turns out that there exists a unique solution to this problem, which moreover can be given a closed-form characterization.

Denote the manager's utility as a function of the report $\hat{x}$ and the true state x by $U(\widehat{x}; x)$, for simplicity just written $U(\widehat{x})$. For the manager's incentive-compatibility constraint to hold, it must be the case that $U(\widehat{x})$ is maximized for $\widehat{x} = x$. The manager

[11] The assumption of no pre-commitment seems plausible for bank or venture capital type of debt contracts, where the relation between the borrower and the lender is of close character, and where concessions made are not necessarily observed by the market. At any extent, we can generate examples where the lender would not wish to commit to being tough with defaults even if such a commitment had zero cost to him.

[12] Deterministic monitoring, assumed in Townsend (1979) and Gale & Hellwig (1985), would imply that an offer slightly less than D would have to be rejected by the creditor, which would not be optimal play the creditor given that the subgame is reached. But if the creditor would accept slightly less than D, the borrower would have incentives to offer even less, and so forth. Hence there cannot exist subgame perfect debt equilibrium under deterministic verification.

has no interest in offering the lender a payment that exceeds D, and the lender's right to demand verification is contingent on offers less than D. Consider therefore values of $\hat{x}$ on the interval $[x_L, D + c_D]$, and let $d := x - \widehat{x}$ be the magnitude of cash flow misreporting. First consider the case $x \in [x_L, D + c_D]$. We then have that,

$$\begin{aligned} U(\widehat{x}) &= Q(\widehat{x})[c_D + d] + [1 - Q(\widehat{x})]0 \\ &= Q(\widehat{x})[c_D + d] \end{aligned}$$

In words, since the manager gets nothing if the creditor refuses to make a concession, the expected utility of the manager after making a report $\hat{x}$ just equals the concession proposal $(c_D + d)$ multiplied by the probability of the creditor accepting the proposal. We now maximize the manager's utility with respect to $\widehat{x}$, where it is assumed that $Q(\widehat{x})$ is differentiable.[13]

$$\frac{dU(\widehat{x})}{d\widehat{x}} = \frac{dQ(\hat{x})}{d\hat{x}}[c_D + d] - Q(\widehat{x}) = 0 \tag{2}$$

For truthful announcement to be optimal, this function must be maximized for $d = 0$, and hence,

$$Q(\hat{x}) - \frac{dQ(\hat{x})}{d\hat{x}} c_D = 0 \tag{3}$$

Solving this differential equation yields,

$$Q(\hat{x}) = K e^{\frac{\hat{x}}{c_D}} \tag{4}$$

Using the corner condition $Q(D + c_D) = 1$, we obtain

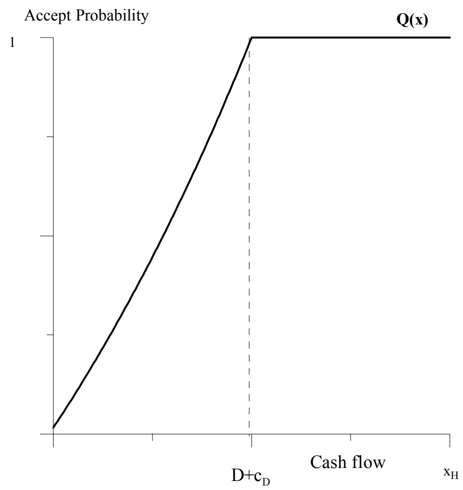
$$Q(\hat{x}) = \begin{cases} e^{-\frac{D + c_D - \hat{x}}{c_D}}, \ \hat{x} \in [x_L, D + c_D] \\ 1, \text{ for } \hat{x} \in [D + c_D, x_H] \end{cases} \tag{5}$$

[13] The equilibrium $Q(.)$ function must be continuous. Were it not for some x, the manager would be made better off by setting the announced x slightly higher than the true x (to thereby pay out only slightly more but have discontinuos jump in accept probability).

Since this accept function induces truth-telling for $x = D + c_D$, it can easily be show that it must also induce truth-telling for $x > D + c_D$. Hence we then have the following result.[14]

Proposition 1 *(Debt) In equilibrium, the manager offers $\tilde{D} = D$ if $x \geq D + c_D$. If $x < D + c_D$, the manager defaults by offering $x - c_D$, and the lender accepts with probability $Q(x) = e^{-\frac{D+c_D-x}{c_D}}$.*

We can represent the proposition in a figure.



The true cash flow of the firm is on the horizontal axis, and the accept probability of the creditor on the vertical axis. The function $Q(x)$ is the equilibrium accept probability, given that the offer from the lender is represented by the function $\tilde{D}(x)$. The accept probability $Q(.)$ is inversely related to the extent of the default $D - x$, which is intuitive because understating the true cash flow must be costly to induce truth-telling.[15] It implies that the lender will be less lenient with firms with large defaults. If we think of the lender accepting the entrepreneur's offer as the firm successfully restructuring its debt out of court and the lender rejecting the offer as the firm going to formal bankruptcy (under

[14]To see that the second order condition for maximum is satisfied, differentiate $U_M(\widehat{x})$ twice with respect to $\hat{x}$, which yields $\frac{Q(d - c_D)}{c_D^2}$ which is clearly negative for $d = 0$.

[15]The intuition for convexity of $Q(.)$ is that it is more tempting for the manager to underreport the cash flow when x is relatively high, so that the steepness of $Q(.)$ must be higher for higher reports.

e.g., Ch. 11), then the proposition implies that firms are more likely to enter formal bankruptcy the larger their default.

Notice that the borrower, expecting the lender to be lenient in certain states by accepting a payment less than the payment contracted upon, ex ante has an incentive to offer a lower repayment than D even though he has sufficient cash to avoid default. In other words, we get strategic defaults in equilibrium for $x \in [D, D + c_D]$.[16]

The leniency on the part of the lender can be seen as an absolute priority violation (AP-violation), since it implies that the borrower receives a positive payoff (with probability $Q(.)$) even though the lender is not paid the full value of his debt contract. In a recent contribution, Bebchuck (2002) studies the effects of AP-violations on the ex-ante risk shifting incentives of borrowers, finding that debt that permits AP-violations induces stronger risk shifting incentives than debt that does not. The effect identified by Bebchuck can be generated in the present setting as well.[17] One important difference between the two setups is that while AP-violations in his setup are imposed exogenously by giving the borrower a fixed fraction of the firm's cash flow/assets in any default states, the AP-violations in the present setting arise endogenously, due to the frictions created by the verification costs.

We should emphasize that the perhaps the most plausible interpretation of the mixed strategy equilibrium played is that the entrepreneur faces a market of possible financiers, and where each financier may play a pure strategy on when to verify (e.g., to verify for any default larger than z, where z is some positive constant), so that the mixed strategy played by the creditor in the model reflects the average behavior played by potential creditors, not the strategy played by each creditor individually. Under this interpretation, the

[16] Esty and Megginson (2001) in an empirical analysis of international lending syndicates argue that syndicates are structured to deter strategic defaults rather than to improve monitoring incentives of lenders.

[17] By showing that debt with AP-violations may induce stronger risk shifting incentives than debt without AP-violations, Bebchuck (2002) identifies an important ex-ante cost of allowing for AP-violations. It may be noted though that this insight is generated by comparing a *riskless* project to that of a risky (less valuable) project. Although using a riskless project as benchmark provides for a clean experiment, the effects on ex-ante risk shifting incentives from AP-violations become more ambigous once the benchmark project is assumed risky as well. In such a case, whether AP-violations will generate greater or less risk shifting incentives will depend on factors such as the amount of debt that the firm issues and the underlying returns generating distribution.

offer function $\tilde{D}(x)$ is a best response to the average or expected play by creditors, not necessarily the best response to the particular creditor played.[18] The same interpretation is applicable to the equilibrium we derive under pure debt and pure equity financing.

We have assumed that verification state payoffs can only depend on x. Alternatively, we could enrich the contractual space by allowing verification state payoffs to depend both on x (resources available) as before, and the report $\hat{x}$ (this assumes that reports are contractible). Specifically, the contract could specify a punishment for the manager if caught lying ($\hat{x} \neq x$), an idea explored by Mookherjee & Png (1989) and Persons (1997). In Appendix D, we consider this possibility and show that this formalization yields qualitatively the same results.

4 Equity

We model outside equity as a contract that gives the investor a fractional right, $\beta \in (0, 1]$, to the firm's cash flow. As with D in the case of debt financing, β will be determined by the funding requirement and the outside investor's participation constraint, but can be viewed as exogenous at this point. The cash flow right associated with equity is supported by an *unconditional* right for the outside shareholder to intervene.[19] We furthermore assume that the verification cost under outside equity is borne by the investor.[20] This assumption implies for example that a shareholder cannot be reimbursed for costs of engaging in a proxy contest.

A strategy by the investor is an offer-function $\tilde{E}(x; \beta)$, for brevity just written $\tilde{E}(x)$, where $\tilde{E} : [x_L, x_H] \rightarrow [0, x_H]$, and $\tilde{E} \leq x$. A strategy for the equity holder is an accept

[18]This is a standard interpretation of mixed strategy equilibria in the game-theoretic literature, see e.g., Rubinstein (1991).

[19]The combination fractional cash flow right and unconditional right to intervene is consistent with equity as observed in practice, and is the same type of approach as e.g., Myers (2000) and Andersen & Nyborg (2001).

[20]With the exception of Proposition 4, our results do not depend on this formulation. For example, letting the insider absorb the verification cost instead gives similar results except that the shareholder is then offered $\tilde{E}(x) = \beta x$ in equilibrium, rather than $\tilde{E}(x) = \beta x - c_E$. The equilibrium accept probability, given β, is independent of who bears the intervention cost c_E ex post. However, the required ownership fraction β in the alternative formulation will be less, since the investor receives βx in equilibrium rather than $\beta x - c_E$. This gives a higher accept probability $P(.)$ and hence lower expected verification costs, but apart from that does not change our results qualitatively.

function $P(\tilde{E})$. As with debt, we consider subgame perfect equilibria of the game between the manager and the equity holder. Given the cash flow right β, and the assumption that the intervention costs is borne by the investor, the investor receives $\beta x - c_E$ in net payoff if he decides to intervene, where c_E is the intervention cost. Analogous to the case of pure debt financing, the outside owner must be indifferent between verifying and not verifying in a subgame perfect equilibrium. Thus, for a given β, we must have that,

$$\tilde{E}(x) = \beta x - c_E \tag{6}$$

which the investor accepts with probability $P(\tilde{E})$. Since the function $\tilde{E}(x)$ is strictly increasing, an offer implicitly defines a reported cash flow, $\hat{x}$. By the Revelation Principle, the question again is whether there exists a function $P(.)$ such that truth-telling is indeed obtained in equilibrium. As for debt, this problem conveniently turns out to have a unique solution, which can be given a closed-form characterization.

Proposition 2 *(Outside equity) In equilibrium, the manager offers the investor $\beta x - c_E$, whereby x is fully revealed, and the investor accepts the manager's offer with probability $P(x) = e^{-\beta \frac{x_H - x}{c_E}}$, $x \in [x_L, x_H]$.*

Proof. See Appendix A. ■

The probability of the outside equity holder intervening is decreasing in the size of the payment that the entrepreneur offers. This is intuitive, the higher the earnings and the higher the dividend payout the less is the chance that shareholders will find it necessary to intervene. Note also that there is a positive probability of intervention for all x, in contrast to what the case is with debt financing.

As can readily be seen, for a given $\hat{x}$, $P(.)$ is decreasing in the outsider's ownership stake β. Intuitively, higher outside ownership increases the potential for the insider to divert cash away from the outsider by under-reporting the true cash flow, which in turn forces the outsider to intervene with a greater probability in order to induce truth-telling. The straightforward implication is that a higher outside ownership implies more active owners.

We may notice that β cannot be arbitrarily small for equity financing to work, because there must be sufficient incentives for the equity holder to intervene after being offered a (low) payment.[21] As we shall see later, this property of equity implies that small projects (a low I) will be 100% debt financed.

We now turn to the case where the firm may be both debt and equity financed.

5 Capital Mix

In considering a mixed capital structure, we take the creditor to be the senior claimant, meaning that the entrepreneur settles his accounts with the creditor before proposing a payout to the outside investor. First, the manager gets funding of an amount I with a fraction α in form of debt and $(1-\alpha)$ in form of equity. The cash flow is then realized and observed only by the manager. Upon observing the true cash flow, the manager offers a debt payment $\tilde{D}$ to the creditor. We assume that the creditor by accepting waives any future rights to the cash flow.[22] The objective of the manager is to pick the financial structure (α) that minimizes expected verification costs, subject to the constraint that the outside investors are willing to participate.

Consider first the case in which the manager does not default on his debt obligation (by offering $\tilde{D} = D$), in which case the creditor has no choice but to accept the offer. After D is paid out to the creditor, the manager proceeds to the shareholder with an offer $\tilde{E}$, where $\tilde{E} \in [0, x - \tilde{D}]$, in which the shareholder accepts with probability $p(\tilde{E})$.[23] If the shareholder accepts the offer, the manager retains $x - D - \tilde{E}$. If the shareholder rejects the offer, the shareholder receives $\beta(x-D) - c_E$, while the manager receives $(1-\beta)(x-D)$. The analysis of this subgame is simplified by the fact that the solution is identical to the

[21] For β small, the condition $\beta x - c_E > 0$ cannot hold for low realizations of x, i.e., the equity holder will not have incentives to monitor when a low cash flow is (truthfully) reported, which is not consistent with an equilibrium. There exists, then, a minimum β, defined as $\bar{\beta} = \frac{c_E}{x_L}$. If a liquidity reserve can be provided ex-ante, by e.g., the outside investors providing more than I, then $\bar{\beta}$ can be decreased, but must still be bounded away from zero.

[22] This is consistent with bankruptcy law as practiced in e.g. the U.S. where repudiation is limited to situations under which the creditor can show that he was coerced to accept the firm's offer (see Berglöf, Roland, and von Thadden (2000) for a discussion).

[23] We assume that the equity holder does not observe $\tilde{D}$, only whether the creditor chose to verify or not. The case where $\tilde{D}$ is observable to the equity holder has qualitatively the same properties as the one in the text, and is considered in Appendix D.

equilibrium of the pure equity financing case, considered in the previous section.[24]

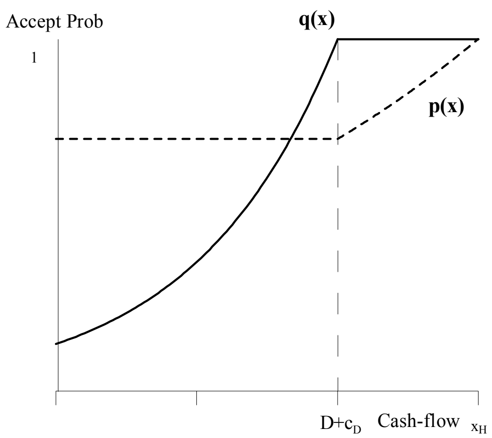
Suppose then that the manager defaults on his debt obligation by putting forth an offer $\tilde{D} < D$. The creditor in this case accepts the manager's offer with a probability $q(\tilde{D})$. If he rejects the offer, he incurs the cost c_D and gets the payout $\min[D + c_D, x]$.[25] If the creditor accepts the manager's offer, on the other hand, leaving a positive amount of c_D for the residual claimants, the manager proceeds to the shareholder with a repayment offer $\tilde{E}$, which the shareholder accept with probability $p(\tilde{E})$. We then have the following.

Proposition 3 *(Capital mix) In equilibrium, if $x \geq D + c_D$, the entrepreneur offers D to the creditor and $\beta(x - D) - c_E$ to the shareholder, which the shareholder accepts with probability $p(x) = P(x) = e^{-\beta \frac{x_H - x}{c_E}}$.*

If $x < D + c_D$, the entrepreneur offers $x - c_D$ to the creditor, which the creditor accepts with probability $q(x) = \mathbf{e}^{\psi(x - D - c_D)}$, where $\psi := \frac{(1-\beta)+\bar{p}\beta}{(1-\beta)c_D + \bar{p}c_E}$, and $\bar{p} = p(D + c_D)$. If the creditor rejects the offer, he verifies and takes the entire cash flow x, which leaves zero for the residual claimants. If the creditor accepts the offer, and thus leaves c_D on the table, the shareholder is offered $\beta c_D - c_E$, which he accepts with probability $\bar{p}$.

Proof. For a full derivation of the equilibrium, see Appendix A. ■

The proposition can be illustrated in the following figure.



[24] With $P(x)$ replaced by $P(x - \tilde{D})$, holding β fixed.

[25] If there is anything left after this, the manager then makes an offer to the outside investor, which again can be accepted or rejected.

The true cash flow of the firm is on the horizontal axis, and the accept probability of the creditor on the vertical axis. The function $q(x)$ is the equilibrium accept probability by the creditor, given that the offer from the lender is represented by the function $\tilde{D}(x)$. The function $p(x)$ is the equilibrium accept probability by the outside investor (recall that he is given an offer only if the creditor has accepted), given that the offer from the lender is represented by the function $\tilde{E}(x)$. The creditor has the role of disciplining the entrepreneur in bad states, and the outside investor has the role of disciplining the manager in good states. For a low x, there is a positive probability of the creditor monitoring, while the probability of the investor monitoring (conditional on the creditor not monitoring) is constant (since the repayments are the same). For a higher x, there is a zero probability of the creditor verifying, and a positive (and decreasing) probability of the investor verifying. Notice that the equilibrium is fully revealing in the sense that the true cash flow is revealed by the offer to the outside investor in the good states, and by the offer to the creditor in the bad states.

Priority violations occur in equilibrium, since in the region $x \in [x_L, D + c_D]$ the lender will accept payments less than D without demanding a verification (with probability $q(x)$), and at the same time the repayment to the investor is strictly positive.[26] As before, strategic defaults occur in equilibrium for $x \in [D, D + c_D]$, by which the manager defaults even though the firm has sufficient cash on hand to pay out the full debt value D.

These results are of some interest, as strategic defaults and violations of priority rules are common explanations for why risk premia on corporate debt significantly exceed those implied by Merton (1974). Strategic defaults occur in the present setting because it is costly for the investor to collect his payment as specified by the contract. For sufficiently low cash flow realizations, though still large enough that default can be avoided, the presence of this cost puts a sufficient wedge between the investor's proper payment under the contract and what the insider is actually willing to offer, thus leading to strategic

[26]If debt contracts could be written such that the creditor would have cash flows rights also after he has decided not to verify (i.e., for the case when the equity holder verifies), it can easily be seen that the equity holders' dominating strategy would be to never verify. In that case, a mixed capital structure cannot occur, and obviously priority violations would be impossible. In practice, such contracts would be costly to enforce.

defaults. As shown by Bergman & Callen (1991) and Mella-Barral & Perraudin (1997), a similar type of effect can occur in symmetric information models, where there is some costs for outside investors to invoke bankruptcy. It may be pointed out, however, that in the present setting, there are AP-violations in the sense that both inside and outside equity receive positive payments even though debt is not paid in full, while the literature on AP-violations (including the papers referred to above) does not distinguish between inside and outside equity.[27]

So far, we have simply assumed that the optimal capital structure is mixed. In the next section, we analyze the optimal capital structure.

5.1 Optimal capital structure

For a given D and β, the expected verification cost is given by,

$$\begin{aligned} V(D,\beta) &= c_D \int_{x_L}^{D+c_D} [1-q(x;.)]f(x)dx + c_E \int_{x_L}^{D+c_D} q(x;.)[1-P(D+c_D)]f(x)dx + \quad (7) \\ & c_E \int_{D+c_D}^{x_H} [1-p(x;.)]f(x)dx \end{aligned}$$

The first two terms is the expected verification costs for low cash flows ($x \in [x_L, D+c_D]$), and the third term is the expected verification cost for high cash flows ($x \in [D+c_D, x_H]$). The objective of the entrepreneur is to pick the α that minimizes this expression, subject to the participation constraints of the investors. Notice that for $\alpha = 0$, i.e., pure equity financing, the first and the second term in (7) drop, and $p(x;.) \equiv P(x;.)$. For $\alpha = 1$, pure debt financing, the second and the third term of (7) drop, and $q(x;.) \equiv Q(x;.)$.

The first observation we can make about optimal capital structure is the following.

Proposition 4 *Firms with a low funding requirement will be financed by debt only.*

Proof. For outside equity holders to have incentives to monitor, they must have a minimum ownership, $\bar{\beta} >> 0$. This implies that the (expected) verification cost is discontinuous in the point $I_E = 0$, where $I_E := (1-\alpha)I$. On the other hand, the expected

[27] The empirical literature on AP-violations (e.g., Franks & Torous, 1989) obtains measures of the sum of AP-violations of internal and external junior claimants.

verification cost is continuous in the point $I_D = 0$. This implies that firms with a low funding requirement (low I) will be 100% debt financed ■

This result is consistent with the idea that to have incentives to monitor managers of firms, shareholders must hold sufficient stakes in the firm to cover their private monitoring costs (see Admati et. al. (1994). In our setting, monitoring is required to induce the manager to pay out funds to investors, and for the threat of monitoring to be credible the shareholder must hold a certain minimum stake in the firm.

Up to now, we have generated results taking β and D as exogenous. To make further headway we need to include the outside investors' participation constraints, and endogenize D and β. For the creditor's participation constraint to hold, his expected payout must equal his financing contribution, αI,[28]

$$\int_{x_L}^{D+c_D} (x - c_D) f(x) dx + \int_{D+c_D}^{x_H} D f(x) dx = \alpha I \tag{8}$$

Notice that the creditor's expected utility is only a function of D, not β, since debt is the senior claimant.[29] For the investor's participation constraint to hold, his expected payout must equal his financing contribution, $(1 - \alpha)I$.

$$\int_{x_L}^{D+c_D} (\beta c_D - c_E) f(x) dx + \int_{D+c}^{x_H} [\beta(x - D) - c_E] f(x) dx = (1 - \alpha) I \tag{9}$$

Combining (8) and (9), we can obtain β as a function of D alone,

$$\beta(D) = \frac{I - \int_{x_L}^{D+c_D} (x - c_D) f(x) dx + \int_{D+c_D}^{x_H} D f(x) dx + c_E}{\int_{x_L}^{D+c_D} c_D f(x) dx + \int_{D+c_D}^{x_H} (x - D) f(x) dx} \tag{10}$$

To find the optimal capital structure, it is more convenient to let D rather than α be the choice variable of the entrepreneur. The first order condition for minimum then becomes,

$$\frac{dV}{dD} = \frac{\partial V}{\partial D} + \frac{\partial V}{\partial \beta} \frac{\partial \beta}{\partial D} = 0 \tag{11}$$

[28]It can easily be verified that the participation constraints must be binding.
[29]Both D and β are functions of α, and hence D and β are related at the ex-ante stage.

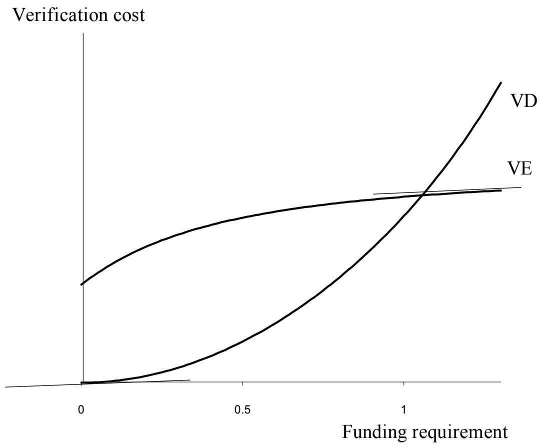
The first and the second partial derivative on the right hand side can be evaluated from $V(D, \beta)$, and the third can be evaluated from (10).[30]

Equipped with these expressions, we have the following.

Proposition 5 *The firm will never be 100% equity financed.*

Proof. Letting D go to 0 in (11) gives a negative expression, as shown in Appendix B. ■

We have established that the firm will be 100% debt financed for a sufficiently low funding requirement, and that the firm will never be 100% equity financed. The basic intuition for these results can be captured by a figure. In the figure, let the cost of verification given pure debt (equity) financing be denoted by VD (VE).



The figure shows VD and VE as functions of the funding requirement, I. For a low funding requirement, pure debt is the optimal financing due to the discontinuity of VE in the point $I = 0$, which arises because a $\beta >> 0$ is required for the equity holder to have incentives to monitor ex-post. That gives intuition for Proposition 4. For a higher funding requirement, VD exceeds VE, and one may think that pure equity dominates. However, having a mix of capital has a lower verification cost than pure equity, because it is on the margin cheaper to issue debt than to issue more equity. This can be captured

[30] Assuming that $\alpha^* \in (0, 1)$, the optimum condition $\frac{dV}{dD} = 0$ will hold for the optimal face value of debt, D^*, and hence the optimal capital structure α^* implicitly, since α is a function of D from equation (8). In other words, D^* uniquely determines α^*.

by comparing the gradient of VD at a low level of debt with the gradient of VE with a high level of equity. That gives intuition for Proposition 5.

Since we know that the optimal capital structure cannot consist of 100% debt, a sufficient condition for a mixed capital structure to occur is that c_E is sufficiently low. However, this is not a very tight sufficient condition, as the optimal debt ratio will be low for a low c_E, as indicated by the following example. The example is 'typical' in that changing parameter and distribution assumptions, we were unable to generate examples that did not have identical qualitative features.[31]

Example 1 $f(x) = \dfrac{1}{x_H - x_L}$, $x_L = 1.2$, $x_H = 4.8$, $c_D = 1$, $c_E = \frac{1}{4}$, $I = 1.5$.

Denoting the optimal choices by a * topscript, we get that for these parameter values (see Appendix B for a derivation), $D^* = .25$, $\beta^* = .55$, $\alpha^* = .16$, $V = .21$. Hence we get a mixed capital structure, where 16% of the capital is raised through issuing equity. By defining the debt ratio of the firm as the (expected) value of debt, αI, divided by the value of the firm, $Ex - V$,

$$g := \frac{\alpha I}{Ex - V} \tag{12}$$

where we get that $g^* = .09$.

By increasing the funding requirement by setting $I = 1.7$ in Example 1, and keeping the other parameters unchanged, we get $D^* = .23$, $\beta^* = .62$, $\alpha^* = .22$, and $g^* = .08$. Hence increasing the funding requirement leads to a lower debt ratio. Decreasing the verification cost, by setting c_E equal to e.g.,$\frac{1}{5}$ in Example 1, we get $D^* = .23$, $\beta^* = .53$, $\alpha^* = .23$, and $g^* = .08$, hence also a decrease in the debt ratio. These two results are as expected given Proposition 4 and Proposition 5.

We can also decrease risk in Example 1, by setting $x_L = 1.5$ and $x_H = 4.5$. In that case we get $D^* = .57$, $\beta^* = .48$, $\alpha^* = .38$, and $g^* = .21$. Hence, when decreasing risk, we get that 38% of the capital is raised through issuing debt, in contrast to 16% before.

[31] The numerical procedure used to generate the example is described in Appendix B. The numbers are generated in Maple V, and the worksheets are available from the authors.

This result is consistent with empirical evidence of less risky firms having a higher debt ratio than more risky firms.[32]

We can summarize these numerical findings with the following remark.

Remark 1 *In example 1, the following gives a lower debt ratio,*

i)Increasing the funding requirement

ii)Decreasing c_E

iii)Decreasing risk

6 Concluding Remarks

We have shown that with a simple reformulation of the classic CSV model from Townsend (1979) and Gale & Hellwig (1985), it can tackle several of the criticisms raised against it, such as lack of subgame-perfectness and its inability to encompass both debt and equity contracts. Furthermore, the implications drawn from this reformulation of the model was shown to be consistent with several stylized empirical facts, such as strategic defaults of debt obligations, capital mix, violations of priority rules, and a higher debt ratio for riskier projects. The main conclusion we draw from this is that the CSV model of financial contracting is more flexible and workable as a basic theory of financial contracting than conceived by earlier literature like Hart (1995).

We see several ways to extend the basic model presented in the paper. First, it may be of interest to introduce dynamics in the model, to tackle such issues as dividend policy and delays in debt repayments. A second possible extension would be to study the interaction of investment incentives and capital structure. For example, it can be shown that while debt finance induces the manager to increase the underlying risk of the project, outside equity generates the opposite incentive. Thus, the firm's capital structure will affect the firm's investment incentives both in the type of project chosen and the amount invested relative to first best. A third extension work would be to discuss commitment debt vis-a-vis non-commitment debt. For example, small investors in the securities market may have commitment through their free-rider status, while banks do not. A preliminary

[32]See survey by Harris & Raviv (1991); and for more recent evidence, Fama and French (2002).

result from our analysis of this question indicates that non-commitment (bank) debt dominates commitment (security) debt (given the same project NPV) for projects with a cash flow distribution which is skewed to the left, which is intuitively appealing, as the non-commitment debt would rely on verifying less often in low cash-flow states.

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8 Appendix A

Here we prove Proposition 2, and then prove Proposition 3.

8.1 Proof of Proposition 2

For the manager to prefer announcing truthfully, it must be the case that,

$$U_M(\hat{x}) = P(\hat{x})[x - \beta\hat{x} + c_E] + [1 - P(\hat{x})](1 - \beta)x \tag{A1}$$

is maximized for truthful reporting, i.e., $\hat{x} = x$. Differentiating (A1) with respect to $\hat{x}$ and setting $\hat{x} = x$ we obtain the differential equation,

$$P(\hat{x})\beta - \frac{dP(\hat{x})}{d\hat{x}} c_E = 0 \tag{A2}$$

Solving this differential equation yields,

$$P(\hat{x}) = K\mathbf{e}^{\frac{\beta\hat{x}}{c_E}} \tag{A3}$$

By using the corner condition $P(x_H) = 1$, we obtain that the probability of the investor the announcement $\hat{x}$ (with an implied offer $\beta\hat{x} - c_E$ to the investor) equals,

$$P(\hat{x}) = \mathbf{e}^{-\frac{\beta(x_H - \hat{x})}{c_E}}, \; x, \hat{x} \in [x_L, x_H] \tag{A4}$$

8.2 Proof of Proposition 3

For a truth-telling equilibrium to exist, as before it must be the case that,

$$\tilde{D}(x) = \begin{cases} x - c_D \text{ for } x \in [x_L, D + c_D] \\ D, \text{ for } x \in [D + c_D, x_H] \end{cases} \tag{A5}$$

There are two cases of interest, $\tilde{D} = D$ and $\tilde{D} < D$. When $\tilde{D} = D$, the true cash-flow is not fully revealed, and we enter the equity subgame, with the same solution as before, i.e.,

$$p(\hat{x}) = \mathbf{e}^{-\dfrac{\beta(x_H - \hat{x})}{c_E}} \tag{A6}$$

The reason for this is the following. Note that

$$\begin{aligned} U_M(\hat{x}) &= p(\hat{x})[x - D - \beta(\hat{x} - D) + c_E] + (1 - p(\hat{x}))[(1 - \beta)(x - D)] \\ &= (1 - \beta)(x - D) + p(\hat{x})[c_E - \beta(\hat{x} - x)] \end{aligned} \tag{A5}$$

Differentiating with respect to $\hat{x}$ and substituting for $\hat{x} = x$, one obtains the first order condition for truthful reporting,

$$\frac{dU_M(x)}{dx} = p(x)\beta - \frac{dp(x)}{dx}c_E = 0 \tag{A6}$$

Using the corner condition $p(x_H) = 1$ and solving the differential equation, we obtain the $p(\hat{x})$ function from (A6).[33]

Now consider the case $\tilde{D} < D$, which must occur when $x < D + c_D$. There are then two cases, the creditor accepting the offer and the creditor rejecting the offer. If the creditor rejects the offer, both the manager and the shareholder receive zero. If the creditor accepts the offer, there remains c_D of the cash flow, and for the equity holder to be indifferent between accepting and not accepting, the manager must offer him $\beta c_D - c_E$ which, by continuity of $p(.)$ is accepted with probability $p(D + c_D)$. The utility of the manager in this case is

$$U_M(x) = q(x)[(1 - \beta)c_D + \bar{p}c_E] \tag{A7}$$

Suppose now that the manager reports $\hat{x} < x$, with the implied offer to the creditor of

[33] As for pure debt and pure equity financing, it can easily be seen that the second order conditions for maximum hold.

$\hat{x} - c_D$, where $d = x - \hat{x}$. If accepted, the manager is now left with $c_D + d$ and offers the shareholder an amount $\beta c_D - c_E$ (where d is sufficiently large to ensure $c_D + d \geq \beta c_D - c_E$ or $d \geq (1-\beta)c_D + c_E$), which the shareholder accepts with probability $\bar{p}$. The utility of the manager from such misreporting becomes,

$$\begin{aligned} U_M(\hat{x}) &= q(\hat{x})[(1-\beta)c_D + \bar{p}(c_E + d) + (1-\bar{p})(1-\beta)d] \\ &= q(\hat{x})[(1-\beta)c_D + \bar{p}c_E + [(1-\beta) + \bar{p}\beta]d] \end{aligned} \tag{A8}$$

Differentiating with respect to $\hat{x}$ and substituting for $\hat{x} = x$ yields the first order condition,

$$\frac{dU_M(x)}{dx} = -\frac{dq(x)}{dx}[(1-\beta)c_D + \bar{p}c_E] + q(x)[(1-\beta) + \bar{p}\beta] = 0 \tag{A9}$$

Solving then for $\frac{dU_M(x)}{dx} = 0$, using the corner condition $q(D + c_D) = 1$, we obtain

$$q(x) = \mathbf{e}^{\psi(x - D - c_D)} \tag{A10}$$

where $\psi := \frac{(1-\beta)+\bar{p}\beta}{(1-\beta)c_D + \bar{p}c_E}$, as stated in the text.

9 Appendix B

As described in the main text, we can write the expected verification as purely a function of D, by combining equation (11) and equation (14). Differentiating V with respect to D in equation (11) we then get,

$$\begin{aligned} \frac{dV}{dD} &= c_D[1 - q(D + c_D)]f(D + c_D) + \int_{x_L}^{D+c_D} \frac{\partial[1 - q(x)]}{\partial D} f(x)dx \\ &\quad - c_E[1 - q(D + c_D)]f(D + c_D) + \int_{D+c}^{x_H} \frac{\partial[1 - p(x)]}{\partial D} f(x)dx \\ &\quad + c_E q(D + c_D)[1 - \bar{p}]f(D + c_D) \\ &\quad + \int_{x_L}^{D+c_D} \frac{\partial[q(x)(1 - \bar{p}]}{\partial D} f(x)dx \end{aligned} \tag{B1}$$

Noting that $1 - q(D + c_D) = 0$, $\frac{\partial[1-q(x)]}{\partial D} = -\frac{1}{c_D}q(x)$, $\frac{\partial[1-p(x)]}{\partial D} = \frac{1}{c_D}(x_H - x)p(x)\frac{\partial\beta}{\partial D}$, we obtain the following first order condition for minimum,

$$\frac{dV}{dD} = c_D \int_{x_L}^{D+c_D} \tfrac{\partial[1-q(x)]}{\partial D} f(x)dx + c_E \int_{D+c_D}^{x_H} \tfrac{\partial[1-p(x)]}{\partial D} f(x)dx + c_E \int_{x_L}^{D+c_D} \tfrac{\partial[q(x)(1-\bar{p}))]}{\partial D} f(x)dx = 0 \tag{B2}$$

where

$$\frac{\partial\beta}{\partial D} = \frac{-\int_{D+c_D}^{x_H} f(x)dx[\int_{x_L}^{D+c_D} c_D f(x)dx + \int_{D+c_D}^{x_H}(x-D)f(x)dx] - \{I - [\int_{x_L}^{D+c}(x-c_D)f(x)dx + \int_{D+c}^{x_H} Df(x)dx]\}\int_{D+c}^{x_H} f(x)dx}{[\int_{x_L}^{D+c_D} c_D f(x)dx + \int_{D+c_D}^{x_H}(x-D)f(x)dx]^2} < 0 \tag{B3}$$

and the second order condition for minimum is $\frac{d^2V}{dD^2} > 0$. Equation (B2) and (B3) define D^* implicitly, and hence the optimal capital structure α^* implicitly, since α is a function of D from equation (12).

The **proof of Proposition 5** proceeds as follows. First, purely for convenience let $c_D = x_L$. Then take D to 0 in equations (B2) and (B3) to obtain,

$$\begin{aligned} \frac{dV}{dD}_{D=0} &= c_E \int_{x_L}^{x_H} \tfrac{\partial[1-p(x)]_{D=0}}{\partial D} f(x)dx \\ &= \frac{c_E}{c_D}\int_{x_L}^{x_H}(x_H - x)P(x)\frac{\partial\beta}{\partial D}_{D=0} f(x)dx \end{aligned} \tag{B4}$$

This expression is negative, since $\frac{\partial\beta}{\partial D}_{D=0} = -\frac{Ex-I}{Ex^2} < 0$.

10 Appendix C: Alternative contracts

Here we consider the possibility that verification state payoffs can be made conditional on both the true cash flow, x, and the announced cash flow $\hat{x}$. Specifically, to obtain truth-telling in the cheapest possible way, for any x, we consider the maximum penalty for false reports, which is to punish such that $U_M(x) = 0$.

10.1 Debt contracts

We suppose that contracts specify the payoffs to the manager in the verification state as,

$$U_M(x, k|\text{verification}) = \begin{cases} x - \min(D, x),\ k = x \\ 0,\ \text{for } k < x \end{cases} \tag{C1}$$

where k is the reported x. Notice that this contract may imply a payout to the creditor higher than D (in the case where x is sufficiently high, and $k \neq x$).[34] We first consider the incentives for truth-telling for $x \in [x_L, D + c_D]$. The utility from truth-telling becomes simply,

$$U_M(x) = Q(x)c_D \tag{C2}$$

The utility from announcing k, where $k < x$,

$$U_M(k) = Q(k)(x - k + c_D) + (1 - Q(k))0. \tag{C3}$$

To obtain truthful reporting,

$$U_M(x) - U_M(k) = Q(x)c_D - Q(k)(x - k + c_D) \geq 0,\ k \leq x,\ x \in [x_L, D + c_D] \geq 0 \tag{C4}$$

This is the same expression as in the original setup, and hence we obtain that for truth-telling to occur for x on the interval $[x_L, D+c_D]$ we get the same solution as in the original setup. We now consider the incentives for truth-telling when $x \in [D+c_D, x_H]$, and where the announcement lies below this interval.

The utility from truth-telling becomes,

$$U_M(x) = x - D. \tag{C5}$$

[34] If the payout to the creditor is limited to D also when a lie is detected, it can easily be shown that the equilibrium accept function is identical to in the original problem.

The utility from announcing k, where $k < D + c_D$,

$$U_M(k) = Q(k)(x - k + c_D) + (1 - Q(k))0. \tag{C6}$$

To obtain truthful reporting,

$$U_M(x) - U_M(k) = x - D - Q(k)(x - k + c_D) \geq 0,\ k < D + c_D < x \tag{C7}$$

Solving for $Q(k)$ we obtain,

$$Q(k) \leq \frac{x - D}{x - k + c_D},\ k < D + c_D < \ x \tag{C8}$$

For every x, this equation defines the set of accept probabilities consistent with truth-telling. The maximum accept probability (which is the relevant to ensure truth-telling in the cheapest possible way) for each x is hence defined as,

$$\frac{x - D}{x - k + c_D},\ k < D + c_D < \ x \tag{C9}$$

As can easily be verified, this function is minimized for $x = x_H$ (for every k).[35] Hence, for truth-telling to occur in the cheapest possible way,

$$Q(k) = \frac{x - D}{x - k + c_D},\ k < D + c_D < \ x \tag{C10}$$

Using the corner condition $Q(D + c_D) = 1$, we obtain the equilibrium accept probability function,

$$Q(x) = \begin{cases} \frac{x_H - D}{x_H - x + c_D},\ x \leq D + c_D \\ 1, \text{ for } x > D + c_D \end{cases} \tag{C11}$$

This function is continuous, increasing, and convex, and takes on the value 1 for $x = D + c_D$. In other words it has the same qualitative properties as the $Q(.)$, function derived in the main text.

[35] Formally, $\frac{x_H - D}{x_H - k + c_D} < \frac{x - D}{x - k + c_D}$, $k < D + c_D < x, x_H$.

10.2 Equity contracts

We consider contracts that are linear in x conditional on truth-telling, but yields $U_M(x) = 0$ in the event of false reports. More specifically, suppose that contracts specify,

$$U_M(x, k|\text{verification}) = \begin{cases} (1-\beta)x, \; k = x \\ 0, \text{ for } k < x \end{cases} \tag{C12}$$

We now derive the accept probability in this case. The utility from truth-telling becomes,

$$U_M(x) = P(x)[x - \beta x + c_E] + [1 - P(x)][x - \beta x] = (1-\beta)x + P(x)c_E. \tag{C13}$$

The utility from announcing k, where $k < x$,

$$U_M(k) = P(k)(x - \beta k + c_E) + (1 - P(k))0 = P(k)(x - \beta k + c_E). \tag{C14}$$

For announcing truthfully to be incentive compatible, it must be the case that,

$$U_M(x) - U_M(k) = (1-\beta)x + P(x)c_E - P(k)(x - \beta k + c_E) \geq 0, \; k < x. \tag{C15}$$

Rearranging,

$$P(x) \geq \frac{P(k)(x - \beta k + c_E) - (1-\beta)x}{c_E}. \tag{C16}$$

For every x, this equation defines the set of accept probabilities consistent with truth-telling. Suppose that truth-telling is hardest to obtain for $x = x_H$ (i.e., has the lowest maximum value of $P(k)$ consistent with truth-telling). Then, imposing the corner condition $P(x_H) = 1$ yields,

$$P(k) \leq \frac{(1-\beta)x_H + c_E}{x_H - \beta k + c_E} \tag{C17}$$

Hence, for truth-telling to occur in the cheapest possible way,

$$P(k) = \frac{(1-\beta)x_H + c_E}{x_H - \beta k + c_E} \tag{C18}$$

In that case, we get an equilibrium accept probability function which equals,

$$P(x) = \frac{(1-\beta)x_H + c_E}{x_H - \beta x + c_E},\ x \in [x_L, x_H] \tag{C19}$$

Notice that this function is increasing and convex, and takes on the value 1 for $x = x_H$. In other words it has the same qualitative properties as the $P(.)$ function derived in the main text.[36]

The question is now under which conditions $P(.)$ defined in (C19) ensures truth-telling for *all* x (or in other words when truth-telling is hardest to obtain for $x = x_H$). In that case $P(.)$ in (C19) is a solution to the problem. We have the following result.[37]

Proposition 6 *For sufficiently small c_E, the $P(.)$ given by (C19) ensures truth-telling in equilibrium for all x.*

Proof. We need to show that for sufficiently small (but positive) c_E, the $P(.)$ function defined by (C19) ensures truth-telling for all x. Letting c_E go to zero in (C15), we obtain that to ensure truth-telling,

$$(1-\beta)x - P(k)(x - \beta k) \geq 0 \tag{C20}$$

[36] Not surprisingly, the $P(.)$ function defined here induces a lower verification cost than the $P(.)$ function derived in the main text. However, we have not taken into account that making announcements verifiable to courts may have some cost.

[37] The problem with generalizing this result to hold for all c_E is that for sufficiently high c_E the function defined by (C19) will not induce truth-telling for all values of x. In particular, there will exist $x << x_H$ such that lying yields a higher payoff then truth-telling. We conjecture that a $P(.)$ function can be defined such that there always exists (truth-telling) equilibria, but this is a rather complex variational calculus problem that lies beyond the reach of the present paper.

substituting in for $P(.)$ implies that,

$$\begin{aligned}&(1-\beta)x-\frac{(1-\beta)x_H}{x_H-\beta k}(x-\beta k) \\ =\;& x-\frac{(x-\beta k)x_H}{x_H-\beta k}=\beta k\frac{x_H-x}{x_H-\beta k}>0;\forall x<x_H,k<x\end{aligned} \tag{C21}$$

By the continuity of $\frac{x_H-x}{x_H-\beta k}$, there exists a strictly positive constant ε, such that $\frac{x_H-x}{x_H-\beta k}>0$ for $c_E\in[0,\varepsilon]$. ■

We can notice that the (expected) verification cost functions for both type of financing in this case is convex, so not surprisingly it can be shown that a mixed capital structure can indeed be optimal also in this modified setup.

11 Appendix D: $\tilde{D}$ is observable to equity holders

In the main text, we assumed that $\tilde{D}$ was unobservable to equity holders (but revealed through the offer). In this appendix, we consider the case where the announcement $\hat{x}$ is observable to the equity holders in addition to the creditors.

11.1 Proof of Proposition 3

For a truth-telling equilibrium to exist, as before it must be the case that,

$$\tilde{D}(x)=\begin{cases} x-c_D \text{ for } x\in[x_L,D+c_D] \\ D, \text{ for } x\in[D+c_D,x_H]\end{cases} \tag{D1}$$

There are two cases of interest, $\tilde{D}=D$ and $\tilde{D}<D$. When $\tilde{D}=D$, the true cash-flow is not fully revealed, and we enter the equity subgame, with the same solution as before, i.e.,

$$P(\hat{x})=\mathbf{e}^{-\frac{\beta(x_H-\hat{x})}{c_E}} \tag{D2}$$

The second case occurs when $\tilde{D} < D$, in which case there remains c_D of the cash-flow after the creditor is paid. In that case, the verification payoff to the equity holder equals $\beta c_D - c_E$, and hence the manager can ensure acceptance with probability 1 by offering $\tilde{E} = \beta c_D - c_E + \varepsilon$, where ε is positive but small. Hence in the equity subgame that follows an accepted offer of $\tilde{D} < D$ to the creditor, the manager offers $\beta c_D - c_E$ (or arbitrarily close) and the equity holder accepts with probability 1.

Let us find the $q(.)$ function that induces truth-telling given $x, \hat{x} \in [x_L, D + c_D)$. If the manager announces $\hat{x}$, the surplus of the manager will be,

$$U(\hat{x}) = q(\hat{x})[c_D(1-\beta) + c_E + d] \tag{D3}$$

Differentiating this expression with respect to $\hat{x}$ and setting $\hat{x} = x$, we obtain the differential equation,

$$q(x) - \frac{dq(x)}{dx}[c_D(1-\beta) + c_E] = 0 \tag{D4}$$

which yields the solution,

$$Ke^{\dfrac{x}{c_D(1-\beta)+c_E}} \tag{D5}$$

We must now determine the integration constant, K. To induce truth-telling, the payoff from truth-telling must be continuous in the point $x = D + c_D$, i.e.,[38]

$$\lim_{x \to (D+c_D)^-} U(x) = \lim_{x \to (D+c_D)^+} U(x) \tag{D6}$$

which implies that,

$$\lim_{x \to (D+c_D)^-} \{q(x)[c_D(1-\beta) + c_E]\} = \lim_{x \to (D+c_D)^+} \{p(x) + (x - D)(1-\beta)\}$$

As argued above, $\lim_{x \to (D+c_D)^+}\{p(x)\} = p(D + c_D)$. Denote $\lim_{x \to (D+c_D)^-}\{q(x)\}$ by $q(D +$

[38]If this condition does not hold, it is easy to see that the manager would have incentives to under- or overreport the true cash flow.

$c_D)^-$. We then substitute into (D6) to obtain,

$$q(D+c_D)^-[c_D(1-\beta)+c_E] = p(D+c_D)c_E + c_D(1-\beta) \tag{D7}$$

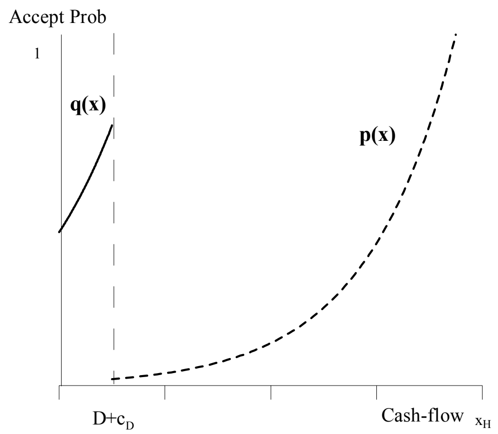
Substituting in for $q(.)^-$ and $p(.)$, we can then determine K,

$$K\mathbf{e}^{\frac{D+c_D}{c_D(1-\beta)+c_E}}[c_D(1-\beta)+c_E] = \mathbf{e}^{-\frac{\beta(x_H-D-c_D)}{c_E}}c_E + (1-\beta)c_D \tag{D8}$$

which gives,

$$K = \mathbf{e}^{-\frac{D+c_D}{c_D(1-\beta)+c_E}} \frac{\mathbf{e}^{-\frac{\beta(x_H-D-c_D)}{c_E}}c_E + (1-\beta)c_D}{c_D(1-\beta)+c_E}$$

Notice that this implies that $q(D+c_D)^- < 1 \neq q(D+c_D) = 1$, in other words is the accept function of the creditor discontinuous in the point $x = D + c_D$. This means that a small default will imply a discontinuous jump (down) in accept probability from 1. The equilibrium then has the following structure, illustrated in a figure.



For a low x, there is a positive probability of the creditor monitoring, while the probability of the investor monitoring (conditional on the creditor not monitoring) is zero. For a higher x, there is a zero probability of the creditor verifying, and a positive (and decreasing) probability of the investor verifying. Hence the monitoring responsibility is completely specialized in equilibrium; the creditor has the role of disciplining the entrepreneur in bad states, and the outside investor has the role of disciplining the manager in good states. We can notice that the probability of the creditor verifying is discontinuous in the point $x = D + c_D$, as in the original setup of Townsend (1979) but now without any assumed commitment power by the creditor.

Priority violations occur in equilibrium, since in the region $x \in [x_L, D + c_D]$ the lender will accept payments less than D without demanding a verification (with probability $q(x)$), and at the same time the repayment to the investor is strictly positive. In fact, since it is known that only c_D remains after the creditor is paid out, the equity holder will accept any offer higher than $\beta c_D - c_E$ with probability 1. As before, strategic defaults occur in equilibrium for $x \in [D, D + c_D]$, by which the manager defaults even though the firm has sufficient cash on hand to pay out the full debt value D.

Let us now consider an example.

Example 2 $f(x) = \dfrac{1}{x_H - x_L}$, $x_L = 1.2$, $x_H = 4.8$, $c_D = 1$, $c_E = \frac{1}{4}$, $I = 1$.

Denoting the optimal choices by a * topscript, we get that for these parameter values, $D^* = .35$, $\beta^* = .34$, $\alpha^* = .35$, $V = .20$, and $g^* = .12$.

By increasing the funding requirement by setting $I = 1.1$ in Example 2, and keeping the other parameters unchanged, we get $D^* = .25$, $\beta^* = .4$, $\alpha^* = .21$, and $g^* = .08$. Hence increasing the funding requirement leads to a lower debt ratio. Decreasing the verification cost, by setting c_E equal to e.g.,$\frac{1}{5}$ in Example 2, we get $D^* = .12$, $\beta^* = .37$, $\alpha^* = .17$, and $g^* = .06$, hence also a decrease in the debt ratio. These two results are as expected given Proposition 4 and Proposition 5.

We can also increase risk in Example 2, by setting $x_L = 1.1$ and $x_H = 4.9$. In that case we get $D^* = .18$, $\beta^* = .38$, $\alpha^* = .18$, and $g^* = .06$. Hence, when increasing risk, we get that 18% of the capital is raised through issuing debt, in contrast to 35% before.