When Does Strategic Debt Service Matter?[1]

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Abstract

Recent work has suggested that strategic underperformance of debt-service obligations by equity holders can resolve the gap between observed yield spreads and those generated by Merton [38]–style models. We show that this is not quite correct. The value of the option to underperform on debt-service obligations depends on two other optionalities available to equity holders, namely, the option to carry cash reserves within the firm and the option to raise new external financing.

We disentangle the effects of the three factors, and characterize the impact of each in isolation as well as their interaction. We find, among other things, that while strategic behavior can increase spreads significantly under some conditions, its impact is negligible in others, and in some cases it even leads to a *decline* in equilibrium spreads. We show that this last apparently paradoxical result is a consequence of an interaction of optionalities that results in a trade-off between strategic and liquidity-driven defaults.

JEL Classification: G13, G35, G33

1 Introduction

Since its introduction by Hart and Moore [18], [19], the notion of *strategic debt-service* has received considerable attention in the finance literature. At its core, strategic debt-service involves a simple idea: when liquidation is costly, it may be possible for equity holders to under-perform on their debt servicing obligations without triggering liquidation, since rejecting the offer and liquidating the firm may leave debt holders even worse off. The idea is an attractive one; it indicates that default may occur not just because the firm lacks adequate cash ("liquidity defaults"), but also because of opportunistic behavior by equity holders ("strategic defaults").

These observations suggest that strategic debt-service makes debt "more" risky and should result in a widening of yield spreads. Recent work in a valuation setting by Anderson and Sundaresan [5] and Mella-Barral and Perraudin [36] appears to confirm this point. The papers each develop cash-flow based extensions of the Merton [38] risky-debt pricing model. Comparing outcomes under strategic and non-strategic debt service using numerical techniques, they find that equilibrium yield spreads are substantially wider in the former case. This leads both papers to conclude, in particular, that strategic debt-service can resolve the widely-documented problem of the underpricing of risky debt associated with the traditional Merton framework.

Two conflating factors, however, mask the exact role played by strategic debt-service in the Anderson/Sundaresan and Mella-Barral/Perraudin analyses. First, neither paper investigates the optimal management of the periodic cash flows generated by the firm. Rather, both require that all cash flows generated in each period be paid out completely to claimholders, either as debt-service to creditors or as dividends to equity holders. In particular, the option to maintain cash reserves to meet future debt-service obligations is unavailable to firms. Second, with regard to raising cash via issue of new equity, the papers make assumptions at opposite ends of the spectrum. Anderson/Sundaresan effectively assume this option is infinitely costly: they prohibit new equity issuance altogether. In contrast, Mella-Barral/Perraudin take it to be costless.

From a theoretical standpoint, these restrictions do not appear to be innocuous; they are also at odds with empirical evidence. Firms in practice often hold substantial reserves of cash; a prohibition on their doing so in the model might overstate liquidity defaults and inflate spreads. Secondly, while an assumption that new equity issuance is infinitely costly appears somewhat excessive, an assumption that it is costless is also questionable. Empirical documentation suggests that such costs are substantial; ignoring them would tend to delay liquidation and bias the value of equity upwards.[1] All of this leads one naturally to ask: When and how much does strategic debt-service really matter? How does its impact depend on cash management policy and equity-issuance costs? These questions form the basis for our analysis in this paper.

We examine a cash-flow based model of firm value akin to that of Anderson and Sundaresan [5] and others. Our model retains the features of earlier work that liquidation is costly and strategic

[1] It also appears inconsistent to assume that liquidation of assets is costly but issue of new equity is not.

debt-service is allowed, but adds two important generalizations. First, it allows the firm to hold cash reserves and carry these forward from one period to the next. Second, it allows for the issuance of new equity to be costly; this cost, like the cost of liquidation, is a model parameter. Taking these costs as given, and anticipating rationally the acceptance/rejection decisions of debt holders in the event of underperformance of debt-service obligations, the firm chooses cash reserve, debt-service, dividend, and equity-issuance policies to maximize equity value.

Our aim is to identify, both theoretically and numerically, the individual impact of each of the three factors highlighted above: (a) cash management policies, specifically, optimal cash management compared to the alternative of "residual" policies, where all cash left over after debt service is paid out as dividends each period, (b) equity-issuance costs, and (c) strategic debt service. More particularly, we seek to understand the *interaction* between these factors, so that we can isolate conditions under which each factor—and especially strategic debt-service—matters.

Expectedly, we find that each factor affects equilibrium debt and equity values, sometimes substantially. For example, requiring residual rather than optimal cash-management policies (i.e., prohibiting cash reserves) generally biases yield spreads upwards, in some cases by more than 200 basis points (for a 10 year bond). Similarly, equity-issuance costs have a very significant impact on equilibrium debt spreads: under optimal cash management, the difference between the two extremes where it is costless and where it is prohibitively expensive can also be 200 basis points and more.

More pertinently, we find that interaction is key: the incremental impact of each factor depends centrally on the others. For instance, the impact of cash management policy depends on equity-issuance costs: residual and optimal cash-management policies create very similar outcomes when the cost of new equity issuance is small, but a substantial difference arises when equity-issuance becomes expensive. Intuitively, with low equity-issuance costs, any cash shortfall can be made up inexpensively by issuing new equity, so optimal cash management is relatively unimportant; but when these costs become large, this alternative to retaining cash becomes unprofitable, creating a significant gap between residual and optimal cash management policies. Similarly, the impact of equity-issuance costs on equilibrum debt values depends on the cash-management policy. We show that if one assumes a residual cash management policy, equilibrium debt values could, counter-intuitively, be *independent* of equity-issuance costs! This is not true under optimal cash management, as was implicitly noted in the previous paragraph.

Finally, the separation of the influence of each of these factors enables us to answer the central question motivating our analysis: when is strategic debt-service important? We find that the answer depends in an essential way on both the costs of equity issuance *and* the cash management policy that is assumed. Specifically, while strategic debt service plays a significant role in determining overall spreads (under either cash-management policy) when the cost of issuing new equity is low, its impact at high costs is typically much diminished and sometimes negligible.

More remarkable, and certainly more counter-intuitive at first sight, is the *direction* of this impact. One might expect strategic debt service to widen yield spreads compared to non-strategic debt service, and, indeed, we show that this does result if one imposes residual cash-management policies. However, for the more relevant case of optimal cash management, we show this is *not* true: while strategic debt-service does widen spreads in this case when equity-issuance costs are low, it could actually *narrow* them when these costs are high! That is, debt is *not* always more risky in the presence of strategic debt service.

The intuition underlying this striking conclusion may be summarized in terms of an interaction of optionalities. The possibility of active cash management gives rise to one option to equity holders, that of retaining cash reserves in the firm as a measure of protection against costly liquidation. The possibility of strategic debt-service creates a second option, that of under-performing on debt-service obligations when this will not trigger liquidation. Now, carrying cash reserves helps the firm avoid costly liquidation in some states, but it has the disadvantage that in those states where liquidation is unavoidable, debt holders have first claim on the firm's assets including these reserves. Strategic debt-service compensates for this partially: it ensures that in the *non-liquidation* states, "excess" reserves (i.e., reserves over the minimum debt-service required to stave off liquidation) accrue to equity holders rather than debt holders. However, if debt-service is non-strategic, these excess cash reserves will continue to accrue to the benefit of debt holders until debt-servicing obligations are fully met.

These statements indicate that the option to carry forward cash reserves is more valuable to equity holders when the option to service debt strategically is also present. Put differently, the firm may find it unattractive to fully exercise its cash reserve option without the underperformance option, i.e., there may be circumstances in which the firm will want to carry *lower* cash reserves under non-strategic debt service than under strategic debt service. These lower level of reserves translate to lower liquidation values and ultimately to *lower* debt values under non-strategic than strategic debt-service. We elaborate on these ideas in Section 5.3 and provide a detailed example there highlighting the interaction of optionalities; the conclusions are further illustrated by the numerical analysis in Section 6 which shows that spreads under strategic debt service could be lower by 40 basis points or more compared to non-strategic debt-service.

Some comments on our analysis are in order before proceeding to the main body of the paper. The introduction of cash reserves and equity-issuance costs results in a manifold increase in the complexity of our model over models which involve residual cash management and zero (or infinite) equity-issuance costs. In particular, a very severe *path-dependence* is injected into the game—at each point in time, the optimal cash management policies of the firm depend on the liquidation value to the debt holders, which in turn depends on the existing cash reserves of the firm, and this last quantity depends on the exact periodic cash flows that have been generated at each previous point in time as well as the equilibrium actions of the various participants at those nodes. All of this makes it extremely difficult to solve for equilibrium values, even numerically, under general debt-structures. However, we show that for the most popular case studied in the literature, that of zero-coupon debt, equilibrium policies *can* be characterized analytically, so the problem *is* amenable to numerical analysis. We focus on this case for the most part in this paper. (Of course, the intuitive arguments presented above are compelling, and the conclusion that strategic behavior need not widen debt spreads should hold more generally.) In the final part of the paper, we present some analytical results and an example with coupon-debt. We show here that the presence of coupons provides yet other optionalities to the equity holder (specifically, concerning the timing of default), and this results in even richer patterns of outcomes.

Some general points of resemblance between our analysis and others in the literature also bear mention. In a paper focussing on the optimal design of debt contracts, Bolton and Scharfstein [11] make the point that there may be a trade-off between liquidity and strategic defaults. Our analysis, conducted in a very different setting from Bolton–Scharfstein, also points to a similar tension: allowing strategic debt-service in our model increases the likelihood of strategic defaults, but because this encourages the firm to carry larger cash reserves, it could lower the likelihood of liquidity defaults. In particular, our analysis implies that the riskiness of debt does not increase solely as a consequence of allowing for additional types of default.

Secondly, that the interaction of optionalities could lead to apparently paradoxical conclusions has also been made in other contexts recently. Myers and Rajan [42] describe a "paradox of liquidity" where greater liquidity (the ability to convert assets to cash) could *reduce* a firm's debt capacity. They attribute this to the fact that a greater liquidity option increases a borrower's optionality stemming from the ability to change his action ex-post. On a similar note, Morellec [39] shows that asset liquidity increases debt capacity only when bond covenants restrict disposition of assets; by contrast, greater liquidity increases credit spreads on unsecured corporate debt. Finally, Titman, et al [47] also find that "deep pocket" borrowers under certain conditions could face greater borrowing costs than do "credit constrained" borrowers.

Finally, our results have implications for empirical research. One comes from the interaction of cash management and equity issuance: firms facing greater equity issuance costs should carry larger cash reserves which should, in turn, help reduce liquidity defaults. Thus, counter to a first impression, firms facing greater equity issuance costs may also face a lower cost of borrowing. A second arises from the interaction of cash management and strategic debt service. As we have seen, for firms that face high equity issuance costs, strategic debt-service can lead to a narrowing of spreads, whereas for firms facing low equity issuance costs, the spread typically widens as a result of strategic debt-service. This suggests that empirical studies attempting to relate the extent of strategic debt-service to yield spreads should control in the cross-section of firms for the relative ease in accessing outside equity. More generally, our results imply that the large number of options equity holders have in practice do not necessarily all work against the interests of the firm's creditors. Empirical work on the agency-theoretic determinants of credit spreads should thus be careful in accounting for such possibilities.

The remainder of this paper is organized as follows. Section 2 describes the related literature.

Section 3 outlines our model while Section 4 describes equilibrium. Sections 5–7 identify and characterize equilibrium properties, both theoretically and numerically. Section 8 concludes. The Appendices contain examples and proofs that are omitted in the main body of the text.

2 Related Literature

Our model is related to three strands of the finance literature, namely, those on the management of corporate cash flows, the costs of equity financing, and the valuation of risky corporate debt. We review all three briefly here.

Management of Cash Reserves

It has been recognized at least since the work of Keynes [26] that there are multiple motives—transactions, precautionary, speculative—driving the demand for and management of cash. Our paper examines an aspect of the precautionary motive: the desire to hold sufficient cash to avoid costly default and/or costly external financing.

Empirical evidence that firms maintain a "reservoir" of cash was first documented by Kalay [25] in his study of dividend constraints and payout covenants designed to mitigate the conflict of interest between equity holders and debt holders. Kalay finds that dividend constraints are not binding and, in fact, firms maintain excess cash reservoirs equal to an average of 11.7% of the firm's market value. John [23] finds empirical support for the precautionary motive linked to liquidation costs, namely that firms with greater direct costs of financial distress hold higher levels of cash in relation to total assets. Opler, et al [44] present a comprehensive investigation of the empirical determinants of cash holdings by firms. Among other things, they find evidence in favor of the precautionary motive linked to external financing costs: namely, that firms with greatest access to capital markets (large and investment-grade firms) tend to hold lower ratios of cash to non-cash assets.

Costs of External Financing

For non-zero reserves of cash holdings to be optimal, raising external funding must be costly (otherwise firms could simply raise cash as needed). Lee, et al [30] document the direct and indirect costs of raising new equity including underwriting costs and spreads, issue expenses, and the underpricing of the issue. They estimate that the *direct* costs average about 11% for initial public offerings and about 7% for seasoned equity offerings. It is safe to assume that the total costs of raising equity are substantial even for large corporations. Altinkilic and Hansen (2000) provide evidence that the cost of external equity may be a monotone *increasing* function in the amount of external equity since the underwriting spread component of external financing costs may be rising with the size of the issue.[2]

While costs of new equity issuance provide a motive for firms to maintain surplus (or "slack") cash reserves, one must trade-off the costs of holding cash versus the benefits. One version of this tradeoff that has been examined in the literature (e.g., Lucas and McDonald [35], Korajczyk, Lucas and McDonald [28], [29]) is where the benefits relate to the ability to fund projects without delay. Our paper examines another aspect of this tradeoff. The costs in our model are that in the event of default, cash reserves accrue to the debt holder; the benefits are, of course, avoiding costly default and costly equity financing.

The Valuation of Risky Corporate Debt

The valuation of risky corporate debt using a contingent-claims framework was first proposed by Black and Scholes [10]. Merton [38] elaborated on these ideas in a simplified framework that assumes zero liquidation costs and that the Absolute Priority Rule or APR holds upon default, i.e., that debt holders are paid in full before equity holders receive anything. Subsequent empirical investigations (e.g., Jones, Mason, and Rosenfeld [24]) revealed that yield spreads implied by the Merton framework are significantly lower than those observed in the market. Moreover, violations of the APR and renegotiation of debt contracts are widely observed in practice, contrary to the model's assumptions (see, e.g., Alderson and Betker [2], Altman [4], Betker [8], Eberhart, Moore and Roenfeldt [13], Franks and Torous [15], [16], Warner [48], and Weiss [49]).

Motivated in part by these empirical shortcomings, several extensions of the Merton framework have been proposed in the literature; an incomplete list includes Acharya and Carpenter [1], Black and Cox [9], Cooper and Mello [12], Geske [17], Ho and Singer [20], Kim, Ramaswamy, and Sundaresan [27], Leland [31], Leland and Toft [32], Longstaff [33], Longstaff and Schwartz [34], Mello and Parsons [37], and Nielsen, Saa-Requejo and Santa-Clara [43]. Among the extensions that have been studied are (i) the introduction of intermediate or periodic cash flows as the source of firm value, which enables a distinction between default and liquidation, (ii) admitting costs of liquidation, (iii) allowing for renegotiation of contract terms between equity and debt holders, (iv) modelling the violation of the APR, mainly as a consequence of renegotiation in the event of default, and (v) determination of the optimal capital structure of the firm by trading off the tax benefits of debt with the liquidation costs incurred in default (or with the agency costs of debt).

The issue of strategic debt-service which concerns us in this paper was first studied in the context of a valuation model by Anderson and Sundaresan [5] and Mella-Barral and Perraudin [36]. Similar ideas are examined in Anderson, et al [6], Fan and Sundaresan [14], Huang [21] and Pan [45]. Fan and Sundaresan [14] also address the impact of optimal cash-management, but

[2] Corporate finance theory has also pointed out an indirect source of costs in external financing: that imposed by agency costs and information asymmetry (e.g., Jensen and Meckling [22], Myers [40] and Myers and Majluf [41]).

there are three important differences between their paper and ours. First, the issue of strategic vs non-strategic debt-service is not a central concern of their analysis. Second, their paper takes new equity-issuance to be a costless process, so the impact of potential costs in this action are not examined. Third, their model does not allow the firm to retain the cash as reserves; rather they require that retained cash be reinvested in the firm, thereby changing its scale (they presume a constant-returns-to-scale technology). This results in a very important analytical simplification: the current cash flow of the firm acts as a sufficient statistic for describing the state of the world at each point. The difference in approaches is non-trivial: we show in an example in Appendix A.4 that the implications of our model are very different from that of Fan and Sundaresan.

3 The Model

We consider a discrete-time setting with time periods indexed by $t = 0, 1, 2, \ldots$ The riskless rate of interest per period is denoted ρ and is taken to be a constant. It is also assumed that markets are free of arbitrage, so there exists an equivalent martingale measure Q with respect to ρ. All stochastic processes and expectations defined in the sequel are with respect to Q.

At the center of our model is a group of homogeneous "equity holders" who have access to a project that will generate cash flows $\{\widetilde{f}_t\}$ into the indefinite future. The present value at time t, of all current and future cash flows is denoted by V_t. Letting $\beta = (1+\rho)^{-1}$, we have

$$V_t \;=\; f_t + \beta\, E_t[V_{t+1}]. \tag{3.1}$$

The project is partly financed by debt raised from a single homogeneous group of creditors. The debt contract has maturity T and calls for the payment of an amount c_t in each period t up to T. The contract also provides for a contingent transfer of control rights or "liquidation" in the event that contractual obligations are not met in any period; this is at the option of the debt holders. Liquidation may not be a costless process. We will denote by $L(A)$ the cost of liquidating the firm given the value of its assets is A. To impose limited liability, it is assumed that $L(A) \leq A$.

A central innovation in our paper concerns the treatment of cash flows. It is typical in the literature (see, e.g., Anderson and Sundaresan [5] or Mella-Barral and Perraudin [36]) to assume that at each t, any residual amount remaining from the cash flow f_t after meeting debt service must be paid out as dividends to the equity holders (we call this a *residual cash-management policy*). In particular, the firm may not retain cash to meet future debt-service obligations. Our paper separates debt-service and dividend policies. We assume that at each t, any cash left over after debt-service may be divided in any way between dividends paid to equity holders and cash retained by the firm as reserves. Retained cash is taken to be invested at the riskless rate ρ. We will denote by ϕ_t the retained cash available to the firm entering period t. Thus, the time-t value of the firm's reserves and cash flows prior to any payouts or cash injections equals $V_t + \phi_t$.

Our model also allows equity holders to raise additional cash in any period through the issue of new equity. In contrast to much of the existing literature, we allow for this to be a costly procedure: we assume that if the firm issues an amount e of new equity at any point, it incurs a cost of $m(e) \geq 0$, where $m(\cdot)$ is a non-decreasing function (thus, the *net* amount of cash raised is $e - m(e)$). This structure nests as special cases the settings of Leland [31] and Mella-Barral and Perraudin [36] where new equity issue is assumed costless ($m(\cdot) = 0$); and Anderson and Sundaresan [5] where new equity issuance is taken to be prohibitively expensive ($m(e) = e$).

Equityholders choose debt-service and dividend and equity-issuance policies to maximize the value of equity given debtholder behavior. Debtholders choose acceptance and rejection policies taking as given the equity holders' chosen cash-management and equity-issuance policies. Equilibrium arises when these strategies are mutual best-responses. A characterization of equilibrium behavior and analysis of equilibrium debt and equity values is the subject of the remainder of this paper. We begin with a more formal description of the equilibrium process.

4 The Structure of Equilibrium

The structure described below is an extensive-form game of perfect information. Subgame-perfect equilibria may be identified by backwards induction in the usual manner begining with date T.

Equilibrium in Period T

Suppose the maturity period T has been reached and the firm's performance on its debt-service obligations has not provoked liquidation thus far. At T, given the cash reserves ϕ_T, the period–T cash flow f_T, and the amount due on the debt c_T, the equity holders select a debt service amount ξ_T. If the offer meets the contractual obligation, i.e., $\xi_T \geq c_T$, debt holders accept payment and the game effectively ends. However, if $\xi_T < c_T$, the firm is technically in default and debt holders must decide whether to accept it (and receive ξ_T) or force liquidation of the firm. We discuss each of these possibilities and its consequences in detail below.

First, suppose $\xi_T < c_T$ and the debt holders reject the offer. In this event, the firm is liquidated. The value of the firm's assets at this point is $A_T = V_T + \phi_T$ (the value V_T of current and future periodic cash flows plus the cash reserves ϕ_T entering period T). Thus, its post-liquidation value is $A_T - L(A_T)$. Let D_T^L denote the amount recovered by the debt holders out of this post-liquidation value.[3] We make the natural assumption that $0 \leq D_T^L \leq (V_T + \phi_T - L_T)$. Equityholders then receive the remaining portion, so equity value in liquidation is $[V_T + \phi_T] - L_T - D_T^L$.

[3]A common practice in the literature is to assume that the Absolute Priority Rule (APR) holds in the event of liquidation: debt holders are given first claim on the post-liquidation value of the firm. We will make a similar assumption in our examples and numerical analysis.

It is immediate that ξ_T will be accepted if and only if $\xi_T \ \geq \ D_T^L$. Now, conditional on making an acceptable offer, it is in equity holders' interest to make the smallest acceptable offer, which is D_T^L. If the available cash $\phi_T + f_T$ suffices for this purpose, it is clearly optimal for equity holders to make the offer $\xi_T = D_T^L$ and for debt holders to accept. The equity holders then decide (i) how much equity e_T (if any) is to be raised in period T, and (ii) the level of dividend service δ_T. Given these choices, the residual cash in the firm entering period $T+1$ now resolves as

$$\phi_{T+1} \ = \ (1+\rho)[\phi_T + f_T + e_T - m(e_T) - \xi_T - \delta_T]. \tag{4.1}$$

Since $m(e) > 0$, it is never optimal to raise equity if the available cash ($\phi_T + f_T$) is sufficient to make the debt service payment (D_T^L). Thus, $e_T = 0$ in this case and the time–T value of debt (denoted V_T^D) is the liquidation value D_T^L; and the time–T value of equity (denoted V_T^E) is the residual value of the firm's assets:

$$V_T^D \ = \ D_T^L \qquad\qquad V_T^E \ = \ V_T + \phi_T - D_T^L \tag{4.2}$$

If the available cash is insufficient to make a payment of D_T^L, then the difference $D_T^L - \phi_T - f_T$ must be raised via issuance of new equity. However, this could itself be a costly process, so it may not be profitable for equity holders. Specifically, let $e_T^{\min}$ denote the minimum amount of cash that must be raised in the form of new equity if liquidation is to be avoided:

$$e_T^{\min} \ = \ \min\{e \geq 0 \mid \phi_T + f_T + e - m(e) \geq D_T^L\}. \tag{4.3}$$

Intuition suggests that equity holders should issue this new equity only if the costs of doing so are less than the liquidation costs L_T. We show this to be true:

Proposition 4.1 *Suppose $f_T + \phi_T < D_T^L$. If $m(e_T^{\min}) \leq L_T$, it is optimal for equity holders to avoid liquidation by raising new equity. The time–T values of debt and equity are*

$$V_T^D \ = \ D_T^L \qquad\qquad V_T^E \ = \ V_T + \phi_T - D_T^L - m(e_T^{\min}) \tag{4.4}$$

If $m(e_T^{\min}) > L_T$, then it is optimal for equity holders to have the firm liquidated, and we have

$$V_T^D \ = \ D_T^L \qquad\qquad V_T^E \ = \ V_T + \phi_T - D_T^L - L_T \tag{4.5}$$

Proof Suppose the equity holders raise a level of equity $e_T \geq e_T^{\min}$. Then, the cash available to the firm is $\phi_T + f_T + e_T - m(e_T)$. Net of debt-service D_T^L, therefore, the value of the firm's assets is $V_T + \phi_T + e_T - m(e_T) - D_T^L$. Of this quantity, e_T represents the value of the new equity raised in period T, so the value of the "existing" equity is simply

$$V_T + \phi_T - m(e_T) - D_T^L. \tag{4.6}$$

Since $m(\cdot)$ is a monotone increasing function, it is apparent from (4.6) that *conditional* on raising enough new equity to avoid liquidation, the value of existing equity is maximized by raising the minimum required amount $e_T^{\min}$, so that existing equity has the time–T value

$$V_T + \phi_T - m(e_T^{\min}) - D_T^L. \tag{4.7}$$

Alternatively, if equity holders do not raise new equity, liquidation results and equity value is

$$V_T + \phi_T - L_T - D_T^L. \tag{4.8}$$

A comparison of (4.7) and (4.8) establishes the required result. □

Remark: The Importance of Cash Reserves

The critical difference between optimal and residual cash-management policies can be illustrated using the time–T payoffs derived above. For specificity, we focus on the case where new equity is prohibitively costly to issue. In this case, depending on the cash available entering period T, payoffs are given by either (4.2) or (4.5). The discussion easily extends to the general case.

A comparison of (4.2) and (4.5) reveals that time–T equity value V_T^E has a discontinuity at the point where $\phi_T + f_T = D_T^L$. Specifically, note from (4.2) and (4.5) that, except at the point of discontinuity, V_T^E has a slope of at most $+1$ in ϕ_T; in words, retaining an extra dollar in the firm today results, in present value terms, in a gain in equity value of at most a dollar. At the point of discontinuity, however, equity value registers a jump *increase* of L_T, representing liquidation cost "savings" from having enough cash to meet minimum debt-service requirements.

A policy of retaining more cash in the firm at time $T-1$ as reserves may thus be worthwhile if it avoids liquidation in some state. *Ipso facto*, it also reduces the probability of liquidation in equilibrium compared to a residual dividend policy. Indeed, it is easy to see that the *only* situations in which it is optimal to pay out all surplus cash as dividends in period $T-1$ are where either (i) there are no liquidation costs, or (ii) new equity issuance is costless.

Equilibrium in Period $T-1$

Now, consider period $T-1$, and define ϕ_{T-1}, f_{T-1} and ξ_{T-1} in the usual way. If $\xi_{T-1} \geq c_{T-1}$, the offer is necessarily accepted; if not, it may be rejected at the debt holders' option. In the latter case, the firm is liquidated and the debt and equity values resolve as

$$V_{T-1}^D \;=\; D_{T-1}^L \qquad\qquad V_{T-1}^E \;=\; V_{T-1} + \phi_{T-1} - D_{T-1}^L - L_{T-1} \tag{4.9}$$

If the offer is accepted, equity holders must then decide on (i) how much cash e_{T-1} is to be raised through the issue of new equity, and (ii) the dividend payout δ_{T-1} in period $T-1$, exactly as in the case of period T described above. If the firm is unable to raise enough cash to meet the promised payment, then liquidation results and the parties receive the payoffs (4.9). Otherwise, the proposed payments are made, and debt and equity values are realized as

$$V^D_{T-1} \; = \; \xi_{T-1} + \beta \, E_{T-1}[V^D_T] \qquad\qquad V^E_{T-1} \; = \; \delta_{T-1} + \beta \, E_{T-1}[V^E_T] \tag{4.10}$$

where $E_{T-1}[\cdot]$ denotes expectation (under Q) of time–T values conditional on all information at the end of period $T-1$, including the choices of ξ_{T-1}, δ_{T-1}, and e_{T-1}.

From (4.9) and (4.10), it is optimal for debt holders to accept the offer ξ_{T-1} if and only if the debt value from continuation (4.10) exceeds that from liquidation (4.9). Taking this into account, equity holders pick a debt-service offer, and values for new equity issuance and dividend payout, to maximize the time $T-1$ value of existing equity.

Equilibrium in Periods $t < T-1$

Equilibrium in earlier periods obtains in exactly the same way as in period $T-1$ using an induction argument. This completes the description of the game's equilibrium. $\square$

5 Zero–Coupon Debt

It is obvious from the description above that solving for equilibria of the game under general debt-structures is an extraordinarily complex task. A structure of particular interest, which has been the focus of a number of papers begining with Merton [38] and which does turn out to be analytically and numerically tractable, is *zero-coupon* debt:

$$c_t \; = \; \begin{cases} 0, & t < T \\ \bar{c}, & t = T \end{cases} \tag{5.1}$$

In this section and the next, we characterize equilibrium behavior and equilibrium debt and equity values when debt is of this form. This section presents analytical results and examples identifying important properties of optimal policies and highlighting their impact on equilibrium spreads. In the section following, we illustrate and provide quantitative expression to these results through detailed numerical analyses of the model; this numerical analysis also illustrates sharply the interdependencies between the various factors.

Since a number of results are presented in this section, a roadmap may be of assistance. The results in this section are presented in three parts. Section 5.1 first looks at the role of the cash-management policy. It characterizes optimal cash-management policies and describes the nature of the bias introduced into equilibrium valuations by considering residual, rather than optimal, policies. Section 5.2 then looks at the role of equity-issuance costs. It describes conditions under which interim equity-issuance is and is not optimal, and discusses the complex nature of the impact of equity-issuance costs on equilibrium values and yield spreads. Finally, Section 5.3 examines the role of strategic debt-service in determining equilibrium values. It shows that under residual cash management, strategic debt service always results in larger yield spreads than non-strategic debt service, but this is not true if cash management is optimal.

5.1 The Optimal Cash-Management Policy

Our first two results in this section, Propositions 5.1 and 5.2, characterize optimal cash management policies. The third result, Proposition 5.3, uses this characterization to identify special conditions under which residual cash management is also optimal. If these conditions are not satisfied, Example 5.4 shows that residual cash is not optimal in general; a similar, but more subtle, point is made by Example A.1 in the Appendix. Finally, Proposition 5.5 summarizes the nature of bias introduced into the valuation problem by assuming residual, rather than optimal cash-management policies. The exact quantitative nature of the bias as well of the optimal size of cash reserves depends on the problem's other parameters such as equity-issuance costs. The numerical analysis of Section 6 highlights these dependences and demonstrates the quantitative impact of these results in a valuation setting.

With zero-coupon debt, there is no debt-service required in any period up to and including $T-1$. Thus, the cash management policy is equivalent to determining the quantum of dividends to be paid at any point. Our first result shows that the optimal cash-management policy up to period $T-2$ has a very simple form:

Proposition 5.1 *With zero-coupon debt, it is optimal to pay no dividends up to and including period $T-2$.*

Proof See Appendix A.1. □

Proposition 5.1 is particularly useful in the numerical computations of Section 6. To see what drives this result, note that giving up a dollar of dividends in any period $t \leq T-2$ entails no present-value loss to equity holders if the dollar (plus interest at the rate ρ) will be returned as dividends in period $T-1$. Now, the only circumstance in which equity holders will choose to *not* pay out all accumulated cash as dividends in period $T-1$ is where equity value can be enhanced by retaining some cash in the firm because it helps avoid a costly cash shortfall in period T. Thus, the zero-dividend policy either leaves investors no worse off (if all accumulated cash is paid out in period $T-1$) or strictly better off (if the retained cash helps prevent costly liquidation or raising of new equity). Proposition 5.1 follows.

What is the optimal dividend policy in $T-1$? In general, the answer depends on the specific structure of the problem in question (equity-issuance costs, the volatility of the cash flow process, liquidation costs, etc). In one special case, however, the answer is unambiguous:

Proposition 5.2 *Suppose either (a) there are no liquidation costs, or (b) equity may be raised costlessly. Then, it is an optimal policy to pay out all cash reserves as dividends in period $T-1$.*

Proof In either of the cases considered in the statement of this proposition, there is no gain to be made by equity holders in carrying cash from $T-1$ to T, but there is an important possible loss: if liquidation occurs at T, debt holders have first claim on the firm's assets including the cash reserves. Thus, it cannot be optimal to carry cash reserves into T, □

By combining Propositions 5.1 and 5.2, we can now answer one question of interest: when are the residual cash-management policies studied in the literature actually optimal? We have:

Proposition 5.3 *Suppose either (a) there are no liquidation costs, or (b) there are no equity-issuance costs. Then, residual cash-management policies are fully optimal; that is, it is an optimal policy to pay out all cash flows as dividends in each period up to and including $T-1$.*

Proof From Proposition 5.1, it is always optimal to pay no dividends up to and including period $T-2$, and to simply retain all cash flows up to that point as reserves. On the other hand, from Proposition 5.2, it is optimal, in the two situations identified in the statement of the corollary, to carry forward zero reserves from period $T-1$ to period T, i.e., to pay out the entire cash reserves as dividends in period $T-1$. From a present value standpoint, retaining all cash flows in the firm up to period $T-1$ and paying them out as dividends at that point is equivalent to paying out all cash flows as dividends as they occur.[4] □

Of course, in general, equity issuance costs and liquidation costs are non-zero, and one might expect that a zero reserve policy is not then generally optimal. Indeed, it is not difficult to see that there are situations where zero reserves are *strictly* suboptimal for *any* non-zero equity-issuance cost and *any* non-zero liquidation cost.[5] Here is a simple example. The single-period of this example may evidently be seen as the last period of a multi-period model.

[4] Proposition 5.3 actually *overstates* the optimality of residual policies in one sense. With coupon debt, it remains true that residual cash-management policies are fully optimal if equity-issuance costs are zero. However, this is not true if liquidation costs alone are zero. Section 9 provides an example with coupon debt and positive equity-issuance costs where residual policies are *strictly* suboptimal *even though liquidation is costless*.

[5] Our choice of a discrete-time model for cash management policy accords well with reality on two counts. First, firms in practice pay dividends only discretely. Second, extraordinary dividends just prior to declaring bankruptcy are illegal in most countries under the principle of *fraudulent conveyance* (e.g., Baird, 1998: "Transfers made and obligations incurred with the intent to delay, hinder or defraud creditors are fraudulent and void as against creditors."). On the other hand, a continuous-time setting for cash management would cause violations on both counts, the first by definition and the second because it would evidently facilitate exhaustion of cash reserves through extraordinary dividends just before bankruptcy. Note that equityholders in our model may also choose to exhaust cash reserves completely at any node if they wish (e.g., if they feel that the likelihood of bankruptcy

Example 5.4 Consider a firm which is currently in period $T-1$ and has zero-coupon debt with face value $\bar{c}$ outstanding that matures at T. Suppose that: (i) With positive probability, the period–T cash flow f_T will be inadequate by itself to repay the face value $\bar{c}$; (ii) The post-liquidation firm value is sufficiently large compared to $\bar{c}$ that any underperformance in period T will lead to liquidation by debt holders; (iii) Equity-issuance costs and liquidation costs are both strictly positive; and (iv) Current cash reserves (including current cash flow) are large enough that underperformance can be avoided for certain by carrying current cash reserves to period T. Then, it is intuitively apparent that no matter how small equity-issuance costs or liquidation costs are, it is always *strictly* better for equity holders to carry positive reserves into period T and avoid underperformance.

Here is a specific illustration of this scenario. Consider a firm with zero cash reserves entering period $T-1$ and a period $T-1$ cash flow of $f_{T-1} = 5$. Suppose, as in Anderson and Sundaresan [5], that the firm faces a binomial cash flow process: for any t, we have

$$f_{t+1} = \begin{cases} uf_t, & \text{with probability } 1/2 \\ df_t, & \text{with probability } 1/2 \end{cases}$$

where $u = 1.10$ and $d = 0.90$. Suppose also that $\beta = 0.95$ (so the risk-free rate is $\rho = 0.0526$). A simple computation shows that for any t, the time–t present value of all current and future cash flows, V_t, is given by $V_t = f_t/[1-\beta]$. In particular, $V_{T-1} = 100$ and V_T evolves according to the same binomial process

$$V_T = \begin{cases} uV_{T-1}, & \text{with probability } 1/2 \\ dV_{T-1}, & \text{with probability } 1/2 \end{cases}$$

Suppose that liquidation costs are strictly positive: $L(A) > 0$ for all A, and that equity-issuance costs are also strictly positive: $m(e) > 0$ for all $e > 0$. Finally, suppose the firm has debt with a face value of $\bar{c} = 8$ outstanding that is due on date T.

At date T, the present value of the firm's current and future cash flows will be either 90 (if state d occurs) or 110 (if state u occurs). Assume that the debt holders can always recover the amount $\bar{c} = 8$ owed to them by liquidating the firm in period T; given the relative sizes of V_T and $\bar{c}$, this is a very mild condition.

Now, if the firm carries the entire cash flow of $f_{T-1} = 5$ into period T, liquidation can be avoided for certain, and a little computation shows that the present value of equity under

at successor nodes is too high). However, since this choice is made in discrete-time, this creates the intriguing possibility that a firm may have positive cash reserves at the time of declaring bankruptcy. This implication too accords well with reality, the most spectacular case in point being the recent collapse of WorldCom which was estimated to have sufficient cash in hand to meet requirements till mid-2003.

this strategy, viewed from period $T-1$, is

$$\beta\left(\frac{1}{2}\left[uV_{T-1}+(1+\rho)f_{T-1}-\bar{c}\right]+\frac{1}{2}\left[dV_{T-1}+(1+\rho)f_{T-1}-\bar{c}\right]\right). \tag{5.2}$$

On the other hand, suppose the firm carries zero reserves into period T. Then, the firm must raise sufficient equity to pay off the debt or liquidation results. Let m^u and m^d denote the minimum costs, in states u and d, respectively, of raising sufficient equity to pay off the debt. Liquidation is optimal in any state for the equity holders if the cost of liquidation in that state is less than the minimum equity cost that must be incurred. Thus, the value of equity under zero cash reserves is

$$\begin{aligned} f_{T-1} \;+\; \beta\,\Bigg(&\frac{1}{2}\left[uV_{T-1}-\bar{c}-\min\{L(uV_{T-1}),m^u\}\right] \\ &+\frac{1}{2}\left[dV_{T-1}-\bar{c}-\min\{L(dV_{T-1}),m^d\}\right]\Bigg) \end{aligned} \tag{5.3}$$

Since $\beta(1+\rho)=1$, a comparison of (5.2) and (5.3) shows that zero reserves are strictly suboptimal no matter how small the liquidation costs L or the equity-issuance costs m. □

Example 5.4 makes the intuitive point that if liquidation or equity-issuance costs can be avoided for certain by carrying cash reserves, then such reserves should optimally be carried. In the example, only equity values are affected by the cash-management policy; debt holders receive the full face amount due to them. In general, both of these features may not hold. That is, liquidation may not be avoidable in some states, so carrying reserves may change both debt and equity values. Now there is a cost also to carrying reserves: by doing so, the firm raises the liquidation value that can be obtained by debt holders, so increases the reservation value payments it will have to make in the non-liquidation states; while, in the liquidation states, these excess reserves accrue to debt holders. Such a situation involves more subtle trade-offs than Example 5.4, and the optimal level of reserves will typically be strictly positive but less than the maximum feasible. Example A.1 in Appendix A.2 presents a specific instance of such a case, and shows that both debt and equity values are indeed affected by the cash-management policy.

Our final result of this section pertains to the bias introduced into the valuation problem by assuming residual instead of optimal cash management. The terms "higher" and "lower" in its statement are to be taken as referring to the corresponding weak inequalities $\geq$ and $\leq$.

Proposition 5.5 *Let debt have the zero-coupon structure (5.1).*

1. *Suppose there are either (a) no liquidation costs, or (b) no costs of raising equity. Then, equity and debt values are the same under optimal and residual cash-management policies.*

2. *Suppose liquidation costs are strictly positive and issuing new equity is costly. Then, the values of debt and equity under an optimal cash-management policy are higher than under a residual cash-management policy. In particular, equilibrium yield spreads on debt are lower under optimal cash-management than residual cash management.*

Proof Part 1 of Proposition 5.5 follows from Proposition 5.3. To see Part 2, note that equity values will obviously be higher under an optimal cash-management policy, since providing equity holders with additional policy choices cannot lower their equilibrium value (in particular, equity holders can always choose zero reserves). Debt values at maturity will also be higher under optimal cash-management since the presence of cash reserves may benefit debt holders in the event of liquidation (by increasing their receipts from liquidation) and therefore by increasing their reservation values, but it cannot, in any circumstance, hurt them. By backwards induction, the value of debt in each earlier period is also higher under optimal cash-management than under residual cash management.[6] □

Our numerical analysis in Section 6 shows that the bias in equilibrium valuations and yield spreads can be substantial. As can be seen there, residual policies may overstate yield spreads by 200 basis points or more for some parameterizations, though the amount of the bias depends on the equity-issuance cost.

5.2 The Effect of Equity Issuance Costs

We have seen in the previous section that one motivation for carrying cash reserves is to avoid costly liquidation. Since cash can also be carried forward by issuing interim equity (i.e., by issuing equity prior to maturity), the question arises whether such interim issuance can be optimal. We show that the answer turns on the form of the equity issuance cost function $m(\cdot)$. The following proposition provides details:

Proposition 5.6 *If the equity-issuance cost function $m(\cdot)$ is a concave increasing function on $\mathbb{R}_+$, the policy that raises no equity at any interim time-point $t = 0, \ldots, T-1$, is optimal. However, if $m(\cdot)$ is convex, interim equity issuance can be optimal.*

Proof See Appendix A.3. Section A.3.1 shows that for concave $m(\cdot)$, zero interim equity issuance is optimal, while Section A.3.2 provides an example to show that interim equity issuance can be optimal with convex $m(\cdot)$. □

The proof and example in the Appendix are somewhat lengthy, but the result itself has an intuitive underpinning. With a non-decreasing concave cost-of-issuance function, it is better to

[6] Fan and Sundaresan [14] state a similar conclusion: that bond convenants which restrict dividend payouts may enhance ex-ante equity as well as debt values.

raise the equity in one go than to break it up into smaller chunks, since, for any $e_1, e_2 > 0$, we have $m(e_1+e_2) \leq m(e_1)+m(e_2)$. With convexity, this inequality reverses, so it may be cheaper to raise a given level of equity in small bits. Of course, this speaks to only a part of the picture; there are additional complications to be considered. For instance, suppose there is currently adequate value in the firm, so it is possible to raise equity today, but this is not true in some poor states in future where there may be inadequate value in the firm. This uncertainty concerning the feasibility of future equity issuance offers a plausible argument, even under concavity, for interim equity issuance. Weighing against this is the observation that in the event of liquidation in the future, debt holders will obtain the benefits of the cash that was raised. The arguments in Section A.3 take all these costs and benefits into account.

Some common and intuitive specifications for $m(\cdot)$ are concave, such as the affine specification

$$m(e) \;=\; \begin{cases} 0, & \text{if } e = 0 \\ m_0 + m_1 e, & \text{if } e > 0 \end{cases}$$

where $m_0, m_1 > 0$. On the other hand, there is the evidence of Altinkilic and Hansen [3] that suggests that underwriting spreads may be *increasing* in the size of the equity issue, implying a convex specification for $m(\cdot)$ on $e > 0$. (Of course, if there is a fixed cost and $m(\cdot)$ is convex for $e > 0$, then $m(\cdot)$ is neither concave nor convex on $\mathbb{R}_+$, so Proposition 5.6 does not apply.) At present, there appears to be inadequate empirical guidance on the form $m(\cdot)$ should take.

Regardless of its form, however, one can show a counter-intuitive conclusion: that under residual cash-management policies, the equilibrium values of debt and equilibrium yield spreads are *not* affected by equity-issuance costs. (Put differently, creditors of the firm are not affected by the costs equity holders face of accessing one source of cash that could be used to repay the debt.) This result is not, however, true under optimal cash management.

Proposition 5.7 *If the cash-management policy is residual, the equilibrium values of debt are unaffected by the costs $m(\cdot)$ of new equity issuance. Under optimal cash management, however, debt values and yield spreads typically depend on these costs, but possibly in a non-montone way.*

Proof In equilibrium, debt holders under either regime receive their reservation value, i.e., the minimum of what they are owed and the post-liquidation firm value. Under residual cash-management, the firm has (by definition) no cash reserves, so the post-liquidation value of the firm depends only on the current and expected future cash flows. In particular, it is independent of equity-issuance costs; so, therefore, is the debt holders' reservation value.

Under optimal cash-management, however, we have seen that the size of optimal cash reserves depends on equity-issuance costs, since carrying reserves helps avoid, among other things, costly equity-issuance. Thus, the post-liquidation value of the firm depends on equity-issuance costs, and so do equilibrium debt values and yield spreads.

As the numerical analysis in Section 6 shows, the dependence of equilibrium spreads on equity-issuance costs under optimal cash-management is "typically" monotone with spreads decreasing as equity-issuance costs increase. In some cases, however, the dependence is U-shaped with spreads first decreasing, then increasing. □

What drives the complexity in the pattern of dependence of equilibrium spreads on equity-issuance costs under optimal cash management? To avoid costly liquidation, equity holders face two options. They can transport cash as reserves across periods, or they can raise cash by issuing new equity as needed. The more the equity-issuance costs, the greater the incentive to avoid liquidation using cash reserves rather than new equity, so the greater the cash reserves carried. Since debt holders have first claim on these reserves in liquidation, this raises debt holders' liquidation values, hence their payoffs in equilibrium. Thus, spreads will, "in general," fall as equity-issuance costs increase. This is the typical case in the numerical analysis of Section 6.

However, unlike equity, cash reserves have the disadvantage that the size of cash reserves is decided in advance, before the state of the world next period is realized. Consider a situation where even carrying the maximum reserves, liquidation cannot be avoided using just the reserves in some of the states next period. Suppose equity issuance costs are sufficiently high that new equity issuance is also not a profitable way to avoid liquidation. Then, equity holders face the choice that larger reserves enable them to avoid liquidation in more states, but also mean that in the "wrong" states these reserves will accrue to the debt holders first. If the likelihood of the latter set of states is suitably high, low and even zero levels of reserves become optimal. This means *smaller* reserves will be carried than at lower equity costs where new equity issuance could have been used to avoid liquidation profitably in some states. Moreover, as equity costs increase in size, the number of states at which liquidation cannot be avoided also increases, so low reserve levels become more generally optimal. This reduces liquidation payoffs to debt holders, leading to an *increase* in spreads and a U-shaped relationship between spreads and equity-issuance costs.

5.3 The Role of Strategic Debt Service

The results of the last two sections, combined with the numerical analysis of Section 6, show that cash-management policies and equity-issuance costs may each exert substantial influence over equilibrium debt and equity values and equilibrium yield spreads. We now turn to the influence of the third factor: strategic debt service, i.e., the deliberate underperformance on debt-servicing obligations when this can be done without triggering liquidation. Two questions concern us here: (i) When is strategic debt service important, i.e., under what conditions is its effect on equilibrium yield spreads largest?, and (ii) Does strategic debt service always increase yield spreads?

The key variable in answering the first question is the size of equity-issuance costs: strategic debt service has its maximum impact on equilibrium spreads when equity-issuance costs are low, but at high costs, its impact is appreciably smaller, and sometimes negligible. The explanation is simple. For underperformance to be feasible without triggering liquidation, it must be the case that (a) the post-liquidation value of the firm is less than the face value of debt due, and (b) the firm has, or can raise, the cash required to make this minimum payment of the post-liquidation value. Condition (a) is exogenous to the firm's actions. Now, the higher are equity-issuance costs, the harder it is for the firm to raise adequate cash by issuing equity and meet Condition (b). In the limit when equity-issuance is infinitely expensive, strategic debt-service becomes feasible only if current cash flow and cash reserves are together larger than the post-liquidation value of the firm. In particular, under residual cash-management, this requires just the *current* cash flow to exceed the post-liquidation value of the firm; this is very unlikely for reasonable parameter values, and, as a consequence, strategic and non-strategic debt service become almost equivalent.[7] All of these contentions are borne out in the numerical analysis in Section 6.

The following result sums up the answer to the second question: does strategic behavior always increase spreads? Its answer—yes, at low equity-issuance costs, but not always if cash managment is optimal and equity-issuance costs are high—acts as a qualifier on the claim that strategic debt-service can resolve discrepancies between observed spreads and those predicted via models such as Merton's [38]. Example 5.9 which follows the proposition makes transparent the interaction of optionalities that could lead to *lower* spreads under strategic than non-strategic debt service.

Proposition 5.8 *If cash management is residual, then strategic debt-service always leads to wider spreads than non-strategic service (though the difference may be negligible at high equity-issuance costs). However, under optimal cash management, strategic debt-service may actually lead to* lower *spreads than non-strategic debt service when equity-issuance costs are high.*

Proof Suppose cash-management is residual, so debt claims are always paid out of current cash flows and equity-issuance. Then, in any terminal state, the payoff to debt holders under strategic

[7] These conclusions support and elaborate on Mella-Barral and Perraudin's [36] finding that strategic debt-service has a large impact under *zero* equity-issuance costs and residual cash management; however, they are in apparent conflict with Anderson and Sundaresan [5] who report the same finding in a model with *infinite* equity-issuance costs and residual cash-management. The source of the discrepancy is likely the following. In their model description (see, e.g., bottom of p.47 of their paper), Anderson and Sundaresan state that in any period $t < T$, new equity issuance is disallowed so forced liquidation occurs if there is inadequate periodic cash flow to meet the minimum debt-service requirement. However, in an apparent inconsistency, they state on p.45 that on date T, equity holders can choose any debt-service in $[0, V_T]$, which implies that costs of new equity-issuance are zero *at this point alone*. If equity financing is costless in every period, then, of course, strategic debt service will matter for the reasons we have outlined in the text; but a similar effect is to be expected even if it is costless in period T alone, since the only "large" payment in their model (the principal amount) comes due on that date. This is, perhaps, what drives the spreads they find. Note, however, that if equity financing is prohibited in all periods (including T), then *strategic* underperformance will rarely be feasible, as outlined in the text; in this case, if sizeable spreads obtain, they are likely the result of *liquidity* defaults caused by the prohibition on cash reserves.

debt service is less than or equal to what they would receive under non-strategic debt service, so equilibrium debt values are lower, and spreads higher, under strategic debt service.

Now suppose cash management is optimal. Then, in any terminal state, the reservation value of debt holders depends also on the cash reserves held by the firm at that point, as does their payoff if liquidation ensues. If equity-issuance costs are low, then the reserves do not matter much (as we have seen, residual and optimal policies nearly coincide in this case). At higher equity-issuance costs, however, the size of the reserves is a critical determinant of both debt and equity values. The firm's incentive to carry larger cash reserves is clearly higher if it can ensure that the benefit from these larger reserves does not accrue to the debt holders, i.e., it can underperform on its obligations where feasible. (This is the *interaction of optionalities* mentioned above—the option to carry cash reserves is more valuable in the presence of the option to underperform on debt-service obligations.) Thus, the firm may carry larger reserves under strategic than non-strategic debt-service, leading to the possibility of *lower* debt values under the latter. Example 5.9 which follows this proof provides a concrete instance of such a situation. □

The following example illustrates the interaction of optionalities. In the example, positive cash reserves are optimal under strategic debt service, but under non-strategic debt service, the option to carry reserves loses its attractiveness, leading to zero reserves and lower debt values. Again, the single period of the example can be considered as the last period of a longer model.

Example 5.9 Consider a firm entering period $T-1$ with reserves of $\phi_{T-1} = 25$ that has debt maturing in period T with a face value of $\bar{c} = 25$. Let the discount factor be $\beta = 0.975$ (so the riskless rate of interest is $\rho = 0.0256$). Suppose that the firm faces a binomial cash flow process: at each t, we have

$$\tilde{f}_{t+1} = \begin{cases} uf_t, & \text{with probability } p \\ df_t, & \text{with probability } 1-p \end{cases}$$

where $u = 1.25$, $d = 0.80$, and $p = 4/9$; and that $f_{T-1} = 0.65$. It is easily checked in this setting that for any t, we have $V_t = f_t/[1-\beta(pu+(1-p)d)]$. In particular, $V_{T-1} = 26$, and, of course, V_T obtains from V_{T-1} through the same binomial process as the cash flow process. Finally, suppose liquidation costs are affine, $L(A) = l_0 + l_1 A$, where $l_0 = 20$ and $l_1 = 0.25$; and that equity issuance is prohibitively expensive.

We consider equilibrium under strategic debt service first, then under non-strategic debt service. Suppose the firm carries a cash reserve of ϕ_T into period T. If the state u occurs in period T, the post-liquidation value of the firm is then

$$uV_{T-1} + \phi_T - l_0 - l_1(uV_{T-1} + \phi_T) = 4.375 + 0.75\,\phi_T.$$

If debt holders are offered less than this quantity, they will liquidate the firm. (Note that this reservation value is increasing in ϕ_T—this is one of the costs of carrying cash reserves.)

However, the amount of cash available to the firm in state u is $uf_{T-1} + \phi_T = 0.8125 + \phi_T$. Thus, for successful stategic underperformance in the state u, we must have

$$uf_{T-1} + \phi_T \;\geq\; uV_{T-1} + \phi_T - l_0 - l_1(uV_{T-1} + \phi_T)$$

or $\phi_T \;\geq\; 14.25$. If the equity holders choose $\phi_T = 14.25$ (which is feasible) and the state d occurs, the liquidation value of the firm in this state would be

$$dV_{T-1} + \phi_T - l_0 - l_1(dV_{T-1} + \phi_T) = 6.2875.$$

Since the firm has enough cash to offer debt holders this quantity, liquidation will be avoided in state d too. A little computation shows that this leads to initial (period $T-1$) equity and debt values given by

$$V^E_{T-1} \;=\; 41.07 \qquad V^D_{T-1} \;=\; 9.93 \tag{5.4}$$

Now, there are essentially three alternatives available to equity holders: (i) Take reserves of $\phi^*_T = 14.25$ into period T and avoid liquidation in both states; (ii) take sufficient reserves into period T to avoid liquidation in state d, but accept liquidation in state u, and (iii) take zero reserves into period T and accept liquidation in both states. Alternatives (ii) and (iii) are equivalent in our setting since the minimum reserves to avoid liquidation in state d is also zero. Another simple computation shows that with zero reserves, the initial values of equity and debt are

$$V^E_{T-1} \;=\; 36.92 \qquad V^D_{T-1} \;=\; 1.90 \tag{5.5}$$

Comparing equity values in (5.4) and (5.5) establishes that with strategic debt service, the optimal reserve size is $\phi^*_T = 14.25$ leading to the valuations (5.4).

Now, suppose debt service is non-strategic. Note that this implies only that *strategic* default is not permitted, i.e., that all cash available must go to the debt holders until debt-servicing requirements are fully met. However, *liquidity* defaults may still occur if there is inadequate cash to meet debt-servicing obligations, and in this case debt holders choose between taking all the available cash and liquidating the firm. Now, if the firm takes no reserves into period T under non-strategic debt service, then liquidation will occur in state u. In state d, however, debt holders are better off accepting the entire available cash of $df_{T-1} = 0.52$ than liquidating the firm. This leads to initial equity and debt values of

$$V^E_{T-1} \;=\; 36.64 \qquad V^D_{T-1} \;=\; 2.18 \tag{5.6}$$

As an alternative suppose the firm took a reserve of $\phi_T = 14.25$ to avoid liquidation in both states. In state u, the outcome is identical to that under strategic debt-service: all the available cash goes to debt holders. In state d, the firm has available cash of $df_{T-1} + \phi_{T-1} = 14.77$, while the post-liquidation value of the firm is only 6.2875. Under strategic debt-service, debt holders had only to be offered 6.2875 to avoid liquidation, but under non-strategic debt service the entire available cash reserve of 14.77 accrues to the debt holders. (This is the interaction of optionalities at work—the option to carry reserves forward is more valuable if the option to underperform strategically on debt-service obligations is also present!) As a consequence, debt and equity values under this choice of reserves works out to

$$V^E_{T-1} \;=\; 36.47 \qquad V^D_{T-1} \;=\; 14.53 \tag{5.7}$$

Thus, with non-strategic debt service the optimal choice of reserves is $\phi^*_T = 0$, leading to the debt and equity values (5.6). In particular, reserves are smaller and yield spreads higher under non-strategic debt service than strategic debt service. □

6 Quantifying the Effects

The purpose of this section is to illustrate in a valuation setting the results derived in the previous section. For this purpose, we must make specific assumptions concerning the cash flow process, liquidation costs, etc. These are presented below.

The Framework

We use a binomial framework similar to the one in Anderson and Sundaresan [5], which in turn is a discrete-time version of the geometric Brownian motion process used by Mella-Barral and Perraudin [36], and others. The complete structure is described below.

The Cash Flow Process Cash flows evolve according to a binomial process: if f_t represents the realized cash flow in period t, the distribution of cash flows in period $t+1$ is given by

$$\widetilde{f}_{t+1} \;=\; \begin{cases} uf_t, & \text{with probability } p \\ df_t, & \text{with probability } 1-p \end{cases} \tag{6.1}$$

A simple computation shows that under (6.1), the value process $\{V_t\}$ is given by $V_t = b^{-1} f_t$, where the constant of proportionality b given by $b = [1 - \beta(pu + (1-p)d)]$. Thus, from (6.1), V_t itself follows a binomial process with parameters u and d:

$$\widetilde{V}_{t+1} \;=\; \begin{cases} uV_t, & \text{with probability } p \\ dV_t, & \text{with probability } 1-p \end{cases} \tag{6.2}$$

Liquidation Costs We take the total liquidation cost L to have the affine form $L = \ell_0 + \ell_1 A$, where ℓ_0 is the fixed component of liquidation costs, ℓ_1 the proportional component, and A the total value of the firm's assets. If the firm never carries forward any cash reserves, we have $A = V$, where V is the present value of the current and future periodic cash flows. If cash reserves are allowed and the firm has a current cash reservoir of ϕ, then $A = V + \phi$. Observe that the *effective* post-liquidation value of the firm is $(A - L)^+ = \max\{0, A - L\}$. This ensures that limited liability of equity holders is not violated and is equivalent to the assumption $L(A) \leq A$ made earlier.

Equity–Issuance Costs Equity-issuance costs are also assumed to be proportional: if the firm issues an amount e of equity, the cost it incurs is $m_1 e$, where m_1 is a non-negative constant. Thus, the *net* receipts to the firm from issuing an amount e of equity are $(1 - m_1)e$. Note that under this specification, zero interim equity issuance is optimal (Proposition 5.6).

Payoffs in Liquidation The Absolute Priority Rule will be assumed to hold in equilibrium. If $\overline{c}$ denotes the face value of (zero–coupon) debt, then liquidation results in the debt holders receiving

$$D_T^L \;=\; \min\{\overline{c}, (A - L)^+\}, \tag{6.3}$$

while equity-holders receive the residual amount $\max\{0, A - L - \overline{c}\}$.

The Parameter Values

We choose parameterizations similar to those chosen by other authors (e.g., Merton [38], or Anderson and Sundaresan [5]). The initial firm value is normalized throughout to $V_0 = 100$, and the zero-coupon debt is taken to have a maturity of 10 years. Each period of the binomial tree is taken to be of 6 months or 0.5 year. The constant b that relates V_t to f_t is fixed at 0.025. The interest rate ρ in each period is set to 0.0247 (i.e., the discount rate $\beta = 0.976$) which when compounded yields an annual rate of 5%. The annualized volatility of the $\{V_t\}$ process is set to $\sigma^2 = 0.10$ (this corresponds to an annualized volatility of about $\sigma = 31.6\%$). The sizes of the up and down moves and the probability of the up move, denoted as u, d, and p, respectively, are calibrated to match this annualized volatility subject to the constraints $u = 1/d$ and $b = [1 - \beta(pu + (1-p)d)]$. Concerning the other parameters, we use a total of 54 different configurations:

1. Three values are considered for the variable cost of issuing equity: $m_1 \in \{0, 0.15, 0.99\}$. The extreme values $m_1 = 0$ and $m_1 = 0.99$ correspond, respectively, to the assumptions of Mella-Barral and Perraudin [36] and Anderson and Sundaresan [5]. The middle value of $m_1 = 0.15$ is a more realistic value given empirical evidence.

2. Three values are considered for the value of debt: $\bar{c} \in \{0.25, 0.50, 0.75\}$. Given the initial firm value of unity, these cover the range from relatively safe to high-risk debt.

3. Three values are considered for the fixed cost of liquidation ($l_0 = 0, 0.10, 0.20$), and two for the variable cost ($l_1 = 0.25, 0.35$).

For space reasons, we do not report equilibrium values for all the parameter combinations here. Parameter values for the numbers reported in Tables 1–3 were chosen as a representative subset and, wherever relevant, also to highlight special features of the equilibrium process.

The Impact of Cash Management Policy

Table 1 summarizes equilibrium equity and debt values and equilibrium yield spreads under residual and optimal cash management strategies for a range of parameter values. It is assumed that debt-service is strategic. The right-most columns of the table also summarize the differences in values across the two cash-management policies.

These differences depend on equity-issuance costs. At zero equity-issuance costs, the two strategies generate identical values, but as these costs increase, residual cash management begins to understate the value of debt and equity, and to exaggerate yield spreads.

For example, at $m_1 = 0.15$, equity values predicted by the optimal cash-management model are higher by about 3%–5% than the values predicted by the residual dividends model. Debt values are also typically higher (as, indeed, they must be from Proposition 5.5), but there is greater variation here, with the differences being negligible in some cases and significant in others. This behavior is reflected in the differences in spreads; at $m_1 = 0.15$, the table shows that the spreads between the models can be virtually identical for some parameterizations, but could also be of the order of 30 basis points or more for others.

As equity costs continue to increase, the difference between the models becomes further exaggerated. As equity costs increase, cash reserves become more important as a means of avoiding costly liquidation. Since such reserves are forbidden under residual dividend policies, "too much" liquidation occurs in this case, leading to lower equity and debt values and higher spreads. Table 1 shows that as new equity issuance becomes prohibitively expensive, the difference in equilibrium values could be dramatic: equity values under optimal cash-management could be 20% or more higher than under residual dividends, and spreads could be more than 200 basis points lower than those implied under residual cash-management.

The Impact of Equity-Issuance Costs

The impact of equity-issuance costs can also be seen from Table 1. First, note that under residual cash-management, debt values and yield spreads are independent of equity-issuance costs. The reason for this counterintuitive implication was discussed in Section 5.2. Of course, equilibrium *equity* values are not independent of these costs: they decline as m_1 increases. It is easy to see why: although debt holders receive the same amount regardless of m_1, the cash flow f_t will, in some states, be insufficient to make this payment, and it may be worthwhile for equity holders to raise the balance by issuing equity. The cost of so doing reduces equity value.

In contrast, under optimal cash management, both debt and equity values depend on equity-issuance costs. In some cases, yield spreads alter very little (15 basis points or so) as a consequence of equity-issuance costs, but in other cases, the difference between yield spreads with costless equity issuance and prohibitively costly equity issuance can be over 200 basis points.

Finally, note that under optimal cash management, the dependence of equilibrium yield spreads on equity-issuance costs is often, but not always, monotone. In most cases, yield spreads decrease as equity-issuance costs increase. In other cases, however, spreads first decrease and then increase again as equity-issuance costs increase, i.e., there is U-shaped behavior. The forces driving these phenomena were also discussed in Section 5.2.

Strategic vs Non-Strategic Behavior

The computations in Table 1 assume strategic debt-service by equity holders. Table 2 considers residual cash management and presents the debt and equity values and equilibrium spreads that would result in this case under *non-strategic* debt-service. The right-most columns of the table describe the differences in equilibrium values and spreads between strategic debt-service and non-strategic debt service. Table 3 presents identical information for optimal cash management.

Table 2 shows that under residual cash management, strategic debt service effectively transfers value from debt holders to equity holders. Equity values are invariably higher under strategic debt service, while debt values are either smaller or the same. Reflecting the last point, spreads are always higher under strategic debt service, or at worst are the same as under non-strategic debt service. This difference is largest at low equity-issuance costs, when it can exceed 200 basis points, but is small or negligible at high equity-issuance costs.

Table 3 shows some points of similarity between optimal and residual cash management. Here too, the differences in equity values are always positive, since having an additional option (to underperform on debt-service obligations) cannot hurt equity holders. Once again, also, the differences are maximal at low equity-issuance costs and smaller at higher equity-issuance costs.

However, there is one significant difference between the tables: under optimal cash management, Table 3 shows that the difference in debt values can be *positive* (and the difference in yield spreads *negative*) at high equity-issuance costs, i.e., strategic debt-service can actually lead to higher debt values than non-strategic debt service. Nor is the consequent *reduction* in spreads insignificant: the lower panel of Table 3 shows that it can be of the order of 25–40 basis points.

7 Coupon Debt

The path-dependence feature of optimal policies gets exacerbated by the presence of the interim debt-service requirements when we move from zero-coupon debt to coupon debt. This introduces significant additional complexity into the model and affects the properties of equilibrium in intricate ways. Essentially, the presence of interim debt-servicing requirements creates a third set of optionalities for equity holders—this one regarding the timing of default. To elaborate on the effects this has, consider, for definiteness, equilibrium debt valuations in the case where equity-issuance costs are positive. With zero-coupon debt, Proposition 5.5 established that

1. With zero liquidation costs, equilibrium debt values are *identical* under optimal and residual dividend policies.

2. With positive liquidation costs, equilibrium debt values are higher[8] under optimal dividend policies than under residual dividend policies.

If the debt structure includes interim debt-service requirements, however, Proposition 7.1 below shows that each of these properties is non-trivially affected:

1. With zero liquidation costs, equilibrium debt values are lower (possibly *strictly*) under optimal dividend policies than under residual dividend policies.

2. With positive liquidation costs, equilibrium debt values under optimal dividend policies can be *higher or lower* than under residual dividend policies, depending on the size of liquidation costs and the debt structure in question.

Several aspects of these results bear emphasis. First, even with *zero* liquidation costs, optimal and residual policies can have markedly different implications. Second, it is possible that optimal cash-management policies lead to *lower* debt values and *higher* spreads than residual policies, which is impossible under any circumstances with zero-coupon debt. Third, with positive liquidation costs, the unambiguous relationship of Proposition 5.5 is replaced by ambiguity on the over/under-valuation issue. A complete statement of these implications follows.

Proposition 7.1 *Suppose the debt structure has some interim debt-service requirements.*

1. *If there are no equity-issuance costs, equilibrium debt and equity values are identical under optimal and residual cash-management policies.*

2. *If equity-issuance costs are positive:*

[8] As earlier, the terms "higher" and "lower" always refer to the weak inequalities $\geq$ and $\leq$, respectively.

(a) *If there are no liquidation costs, equity value is higher and debt value lower under an optimal cash-management policy than under a residual policy. These inequalities can be strict.*

(b) *If liquidation costs are non-zero, equity value is higher but debt value may be lower or higher under an optimal cash-management policy than under a residual policy.*

Proof With zero equity-issuance costs, residual cash-management policies are clearly fully optimal as mentioned in Section 5, so assume equity-issuance costs are strictly positive. Now observe that, regardless of liquidation costs, equity value must always be higher under optimal cash-management policies than residual policies since equity holders cannot be worse off from having additional courses of action available to them. Thus, it remains to be shown that (a) with zero liquidation costs, debt values are lower under optimal cash-management than residual cash-management, and (b) with positive liquidation costs, debt values can be either higher or lower under optimal policies than under residual ones.

To see (a), suppose there are no liquidation costs. In this case, the entire initial value V_0 of the firm is divided between equity value V_0^E and debt value V_0^D, since expected future liquidation costs are zero at all points. If V_0^E is higher (as it must be under optimal cash-management policies), then V_0^D must necessarily be lower. That these inequalities can be strict is shown in Example 7.2 below. This establishes Part 2(a) of the proposition.

To see (b), consider first a given coupon structure which is such that under zero liquidation costs, equilibrium equity values are *strictly* higher and debt values *strictly* lower under optimal policies than residual ones. (The specification in Example 7.2 below is an instance.) A simple continuity argument establishes that if liquidation costs are positive but "small," these inequalities will continue to hold. Thus, there exist scenarios where with positive liquidation costs, equilibrium debt values are strictly *lower* under optimal cash-management than under residual cash-management establishing one part of the result.

To see the other part, consider a specification for which under zero-coupon debt, debt value is *strictly* higher under optimal cash-management than residual cash-management. (For instance, one of the specifications in Sections 5–6.) Another continuity argument shows that if a "small" interim coupon is appended to the debt-structure, the inequalities will continue to hold. Thus, it is also possible that with positive liquidation costs, equilibrium debt values could be strictly *higher* under optimal cash-management, completing the proof. □

Intuitively, as in Geske [17], coupon debt may be viewed as a compound option in the hands of equity holders in which the payment of a coupon entitles the option-holder to proceed to the next payment. Viewed in this light, the presence of coupons provides equity holders with additional options with which to effect a transfer of value from debt holders to themselves. Moving from residual to optimal policies enhances the equity holders' ability to exploit this optionality. As a consequence, debt holders may become worse off than under residual cash-management policies. These intuitive arguments lie at the heart of our construction in Example 7.2 below.

Example 7.2 Consider a cash flow process specified by $f_1 = f_2 = f_3 = 1$,

$$f_4 = \begin{cases} 160, & \text{with probability } 1/2 \\ 80, & \text{with probability } 1/2 \end{cases} \tag{7.1}$$

and $f_t = 0$ for $t \geq 5$. To fix ideas, it may help to think of this as a project with a three-period gestation at the end of which it is marketed, and is either a success (resulting in a cash flow of 160) or a failure (a cash flow of 80). The present value of this project is

$$V_1 = 1 + \beta + \beta^2 + \beta^3[(160 + 80)/2]. \tag{7.2}$$

Now consider a coupon-debt structure for this project, where a coupon payment of 2 is due in period 2, and a coupon of 2 and the principal face value of 100 are due in period 4. In terms of our notation, we have:

$$c_2 = 2, \; c_4 = 102, \; c_t = 0 \;\text{ for all } t \neq 2, 4. \tag{7.3}$$

Next, we assume that if liquidation occurs in period t, the debt holders are owed an amount equal to the coupon due in period t plus the face value of the debt (this rule is employed in practice). Thus, for the two periods where default may occur and the debt holders can force liquidation, the amounts owed the debt holders are, in each case, given by $2 + 100 = 102$. Finally, we assume there are no liquidation costs ($L(A) \equiv 0$), and that new equity issuance is prohibitively expensive ($m(e) = e$).

Consider residual cash management first. At $t = 4$, given zero liquidation costs, any underperformance on the debt contract will automatically trigger liquidation of the firm. Thus, debt holders receive either $\min\{160, 102\}$ or $\min\{80, 102\}$. At $t = 3$, there is no coupon due, so the cash flow of 1 is paid out as dividends. At $t = 2$, there is insufficient cash to meet the coupon payment of 2. There are two alternatives facing the debt holders: (a) accept a payment of $\xi \leq 1$ and allow the firm to continue, or (b) liquidate the firm. In the former case, the value to the debt holders is

$$\xi + \beta^2[(\min\{160, 102\} + \min\{80, 102\})/2] = \xi + 91\beta^2. \tag{7.4}$$

In the latter case, the value of the firm is $1 + \beta + \beta^2[(160 + 80)/2] = 1 + \beta + 120 \cdot \beta^2$. Thus, the value to the debt holders from liquidation is

$$\min\{1 + \beta + 120\beta^2, 102\}. \tag{7.5}$$

It is easily checked that the difference between (7.5) and (7.4) is strictly positive for all $\beta \in (0,1)$ and $\xi \in [0,1]$. It is immediate that debt holders will always choose liquidation in period 2, so debt and equity values resolve as

$$\overline{V}^D = \beta \min\{1 + \beta + 120\beta^2, 102\}. \tag{7.6}$$

$$\overline{V}^E = 1 + \beta \max\{0, 1 + \beta + 120\beta^2 - 102\}, \tag{7.7}$$

We turn now to optimal cash management. Carrying out the computations as above shows that the equilibrium strategy for equity holders is to receive no dividends in period 1; to receive all excess cash after paying the coupon as dividends in period 2; to receive a dividend of 1 in period 3; and to receive the residual amount (either $160 - \min\{160, 102\}$ or $80 - \min\{80, 102\}$) in period 4. The values of debt and equity under this strategy are

$$V^D = \beta(2 + 91\beta^2) \qquad V^E = \beta(\rho + \beta + 29\beta^2). \tag{7.8}$$

Inspection of (7.6)–(7.8) makes it immediate that for all reasonable values of β, debt values are significantly higher, and equity values significantly lower, under residual cash management than under optimal cash management. □

It is apparent that numerical analysis in a setting akin to Section 6 is very hard for general coupon debt structures: an analytical characterization of optimal policies appears infeasible in the general case, and numerical estimation in the absence of such a characterization involves searching over *all* possible dividend/debt-service policies at each node. Nonetheless, the results we have presented in this section underscore the importance of optimal cash-management policies and, more generally, the interaction of optionalities that is at the heart of our paper.

8 Conclusions

It has been suggested in recent work that strategic underperformance of debt-service obligations by equity holders raises yield spreads and can resolve the gap between observed yield spreads and those generated by Merton [38]–style models. However, the models offered in support have placed somewhat strong restrictions on two other important "optionalities" available in principle to equity holders: the option to carry cash reserves as protection against costly liquidation, and the option to raise cash by issuing new equity. This makes it hard to separate the effect of strategic behavior from the impact of the other constraints.

Our paper removes these restrictions and disentangles the effect of the three factors, characterizing their individual impact as well as their interdependence. We find that while each factor could have a significant impact on equilibrium, there is a strong "interaction of optionalities" that determines when a factor is important. In particular, we show that strategic debt-service has a large impact on equilibrium when new equity-issuance costs are low, but that at high equity-issuance costs, its impact is much diminished. More strikingly (and certainly more unintuitively at first sight), we show that strategic debt-service could actually *lower* spreads compared to non-strategic debt service.

Our results carry implications for empirical work in this field. For example, they suggest that in the cross section, the effect of strategic behavior on yield spreads should be positive and significant for firms that can access alternative sources of financing cheaply, but that the effects should be less significant, and perhaps even negative, for firms facing high costs in this direction. Thus, controlling for the costs of accessing outside financing, and more generally, taking into account the interaction of optionalities, is important in doing empirical work on the agency-theoretic determinants of credit spreads.

A Proofs & Further Examples

A.1 Proof of Proposition 5.1

The proof uses a backwards induction argument.

Period T In the last period, the equilibrium evidently has the same structure as it does in the general case. Thus, given the cash reserves ϕ_T and the period–T realized cash flow f_T, the equilibrium debt and equity values at T are given by (4.2)–(4.5).

Period $T-1$ Given the cash reserves ϕ_{T-1} entering $T-1$ and the realized cash flow f_{T-1} in that period, the owner–manager must decide on the dividend δ_{T-1} to be paid that period. (There is no debt-service amount to be considered.) Given δ_{T-1}, ϕ_T is determined as

$$\phi_T \;=\; (1+\rho)(\phi_{T-1} + f_{T-1} - \delta_{T-1}). \tag{A.9}$$

The one-to-one relationship between δ_{T-1} and ϕ_T present in (A.9) implies that we could equivalently model the owner as picking ϕ_T directly, rather than δ_{T-1}. Since δ_{T-1} must lie between 0 and $(\phi_{T-1}+f_{T-1})$, the range of feasible values for ϕ_T is $[0, \phi_T^{\max}]$, where $\phi_T^{\max} = (1+\rho)(\phi_{T-1}+f_{T-1})$.

Pick any $\phi \in [0, \phi_T^{\max}]$. Let $E_{T-1}[V_T^E(\phi)]$ denote the expected continuation value of equity given ϕ conditional on information available up to $T-1$. Given ϕ, the dividend in $T-1$ is $\delta_{T-1} = \phi_{T-1} + f_{T-1} - \beta\phi$, so the value of equity in period $T-1$ is

$$(\phi_{T-1} + f_{T-1} - \beta\phi) + \beta E_{T-1}[V_T^E(\phi)]. \tag{A.10}$$

Equityholders pick ϕ to maximize (A.10).

$$V_{T-1}^E \;=\; \max_{\phi\in[0,\phi_T^{\max}]} \{(\phi_{T-1} + f_{T-1} - \beta\phi) + \beta E_{T-1}[V_T^E(\phi)]\}. \tag{A.11}$$

Period $T-2$ Let ϕ_{T-2} be the cash reserves entering period $T-2$, and let f_{T-2} be the realized cash flows in that period. Define $\phi_{T-1}^{\max} = (1+\rho)(\phi_{T-2} + f_{T-2})$. Analogous reasoning to that used above establishes that in period $T-2$, ϕ is chosen to solve

$$V_{T-2}^E \;=\; \max_{\phi\in[0,\phi_{T-1}^{\max}]} \{(\phi_{T-2} + f_{T-2} - \beta\phi) + \beta E_{T-2}[V_{T-1}^E(\phi)]\}. \tag{A.12}$$

We will show that $\phi = \phi_{T-1}^{\max}$ solves this maximization problem. To this end, observe that the value of V_{T-1}^E from (A.11) may be written as

$$V_{T-1}^E \;=\; \phi_{T-1} + \max_{\phi\in[0,\phi_T^{\max}]} \{(f_{T-1} - \beta\phi) + \beta E_{T-1}[V_T^E(\phi)]\}. \tag{A.13}$$

Moreover, since the maximand on the right-hand side does not depend on ϕ_{T-1}, the maximization is affected by ϕ_{T-1} only through $\phi_T^{\max}$. Since $\phi_T^{\max}$ increases as ϕ_{T-1} increases, a larger value of ϕ_{T-1} implies a larger feasible set of actions, and, therefore, a (weakly) larger value of the maximized function. Summing up, the value of equity at $T-1$ has the form

$$V_{T-1}^E \;=\; \phi_{T-1} + G(\phi_{T-1}), \tag{A.14}$$

where $G(\cdot)$ is a non-decreasing function. We use this representation in (A.12). Substituting for V_{T-1}^E from (A.14), we can write the time $T-2$ value of equity as

$$V_{T-2}^E \;=\; \max_{\phi\in[0,\phi_{T-1}^{\max}]} \{(\phi_{T-2} + f_{T-2} - \beta\phi) + \beta E_{T-2}[\phi + G(\phi)]\}. \tag{A.15}$$

The term ϕ can clearly be pulled out of the expectation on the right-hand side. Doing so, and cancelling the common term $\beta\phi$ that results, this yields:

$$V_{T-2}^E \;=\; \max_{\phi\in[0,\phi_{T-1}^{\max}]} \{\phi_{T-2} + f_{T-2} + \beta E_{T-2}[G(\phi)]\}. \tag{A.16}$$

Now, G, as we have already seen, is a non-decreasing function of ϕ. Since the rest of the maximand is independent of ϕ, it follows easily that one solution to (A.16) is to have $\phi = \phi_{T-1}^{\max}$, in particular, to pay no dividends at all in period $T-2$.

Period $t < T-2$ An identical argument to that used for period $T-2$ establishes that it is an optimal policy in each preceding period to not pay any dividends, and instead to hold back the entire cash flow in the firm. This completes the proof of the proposition. □

A.2 The Non-Optimality of Residual Cash Management Policies

Section 5.1 provided an example to show that residual cash-management policies could be strictly suboptimal. In that example, liquidation was avoidable *with certainty* by carrying sufficient cash reserves. The example below makes a more subtle point: even if carrying cash reserves cannot prevent liquidation with certainty (so the extra reserves accrue to debt holders in some states) and even though carrying reserves raises the liquidation value of the firm (and so the reservation level of debt holders), it can still be strictly suboptimal to carry zero reserves.

Example A.1 Consider the same specifications and parameterizations as in Example 5.4, but with the following modifications. Let the face value of debt due in period 1 be $\bar{c} = 15$; let the liquidation costs have the linear form $L(A) = \ell_0 + \ell_1 A$, where $\ell_0 = 80$ and $\ell_1 = 0.50$; and let new equity-issuance be prohibitively expensive, so $m(e) = e$.

The maximum cash reserves the firm can carry into period T is $\phi^{\max} = 5(1+\rho) = 5.263$, while the minimum is, of course, zero. We will show that the optimal reserve is between these extremes. Suppose the firm enters period T with reserves $\phi \in [0, \phi^{\max}]$. As a first step, we calculate the post-liquidation value of the firm given ϕ. In the state u, this is

$$uV_{T-1} + \phi - L(uV_{T-1} + \phi) \;=\; 15 + 0.5\phi \tag{A.17}$$

while in the down-state d, it is

$$dV_{T-1} + \phi - L(dV_{T-1} + \phi) \;=\; 5 + 0.5\phi \tag{A.18}$$

If the state u occurs in period T, the firm has a cash flow of $uf_{T-1} = 5.50$, so has total cash available of $5.50 + \phi$. For any $\phi \in [0, \phi^{\max}]$, this is less than the liquidation value $15 + 0.50\phi$. Thus, regardless of the size of ϕ, the firm will be liquidated in state u, and period–T equity and debt values are realized as

$$V_1^E(uV_{T-1}, \phi) \;=\; 0.5\,\phi \qquad V_1^D(uV_{T-1}, \phi) \;=\; \min[15, 15 + 0.5\,\phi] = 15$$

If the firm enters the state d with reserves of ϕ, it would have total cash of $df_{T-1} + \phi = 4.50 + \phi$. This is greater than the liquidation value $5 + 0.5\phi$ as long as $\phi \geq 1$. Thus, if $\phi \geq 1$, liquidation is avoided, and period–T equity and debt values are realized as

$$V_1^E(dV_{T-1}, \phi) \;=\; 85 \qquad V_1^D(dV_{T-1}, \phi) \;=\; \min\,[15, 5 + 0.5\,\phi] = 5 + 0.5\,\phi$$

On the other hand, if $\phi < 1$, the firm will be liquidated and a simple calculation shows that period–T equity and debt values are

$$V_1^E(uV_{T-1}, \phi) \;=\; 0 \qquad V_1^D(uV_{T-1}, \phi) \;=\; 5 + 0.5\,\phi$$

Viewed from period $T-1$, to take reserves of ϕ into period T, the firm must pay dividends of $5 - \beta\phi$ to equity holders in period $T-1$. Thus, given $\phi \geq 1$, the value of equity at $T-1$ is

$$[5 - 0.95\phi] + \beta[(0.50)V_T^E(uV_{T-1}, \phi) + (0.50)V_T^E(dV_{T-1}, \phi)] \;=\; 45.375 - 0.7125\phi$$

while if the firm chooses $\phi < 1$, the time $T-1$ value of equity is just the period $T-1$ dividend $5 - 0.95\phi$. It follows easily that it is optimal policy to set $\phi^* = 1$, resulting in the values

$$V_{T-1}^E \;=\; 44.90 \qquad V_{T-1}^D \;=\; 9.7375.$$

respectively. In contrast, residual dividend policies would have forced $\phi = 0$, resulting in time–0 equity and debt values of 5 and 9.50, respectively. Thus, undervaluation of both debt and equity at time $T-1$ results. □

A.3 Proof of Proposition 5.6

We first show that if $m(\cdot)$ is concave, the policy of zero interim equity issuance is optimal. Then, we provide an example to show that interim equity issuance can be optimal when $m(\cdot)$ is convex.

A.3.1 The Case of Concave $m(\cdot)$

The proof is by induction.

Period $T-1$ Let V_{T-1} and ϕ_{T-1} be given. Consider a dividend, cash reserve, and equity-issuance policy $\mathcal{C} \equiv (\delta_{T-1}, \phi_T, e_{T-1})$ where (a) δ_{T-1} is the dividend paid out at time $T-1$, (b) ϕ_T is the cash-reserve at time T, and (c) e_{T-1} is the amount of equity raised at time $T-1$. Note that ϕ_T is given by equation (4.1) with ξ_{T-1} set to zero. Note also that if $e_{T-1} > 0$, then $\delta_{T-1} = 0$ since it is never optimal to pay dividends by issuing costly equity.

Given any state ω at time T, and assuming an optimal continuation from T, let the continuation values of debt, liquidation, and period–T equity-issuance costs induced by the policy $\mathcal{C}$ be denoted $V_T^D(\omega)$, $L_T(\omega)$, and $e_T(\omega)$, respectively. Further, denote

$$V_{T-1}^D(\mathcal{C}) \;=\; E_{T-1}[V_T^D(\omega)] \qquad L_{T-1}(\mathcal{C}) \;=\; E_{T-1}[L_T(\omega)] \tag{A.19}$$

These are the current (time $T-1$) expectations of the time T values of debt and liquidation costs, given the current state. We suppress dependence on $\mathcal{C}$ in the sequel. Since equity issuances are simply transfers between claimholders except for the deadweight costs of equity issuance, it follows that the expected value of the firm's equity at time $T-1$ under the policy $\mathcal{C}$ is

$$V_{T-1}^E \;=\; V_{T-1} + \phi_{T-1} - V_{T-1}^D - L_{T-1} - m(e_{T-1}) - \beta\, E_{T-1}[m(e_T(\omega))] \tag{A.20}$$

Let $g(e) = e - m(e)$ be the *net* amount of cash raised after issuing equity of amount e. Now consider an alternative policy at $T-1$ that raises no equity at time $T-1$ and instead raises the amount e_{T-1} at time T. That is, specify the alternative policy by $\hat{\mathcal{C}} = (\delta_{T-1}, \phi_T - \beta^{-1} g(e_{T-1}), 0)$, followed by the raising of $\hat{e}_T(\omega)$ in state ω at time T, where

$$g(\hat{e}_T(\omega)) \;=\; g(e_T(\omega)) + \beta^{-1}\, g(e_{T-1}). \tag{A.21}$$

Note that if δ_{T-1} is feasible under $\mathcal{C}$, it is feasible under $\hat{\mathcal{C}}$ as well. By construction, the amount of cash available in any state ω at time T is the same under $\hat{\mathcal{C}}$ and $\mathcal{C}$; this common amount is equal to $f_T(\omega) + \phi_T + g(e_T(\omega))$. Thus, any time–$T$ dividend policy possible from $\mathcal{C}$ is also possible from $\hat{\mathcal{C}}$. Similarly, note that if $e_T(\omega)$ was a feasible issue in state ω then so is $\hat{e}_T(\omega)$.[9]

[9] To see this, note that feasibility of $e_T(\omega)$ implies that $V_T(\omega) + \phi_T + g(e_T(\omega)) \geq V_T^D(\omega)$ so that equity value is non-negative after equity issuance. Then, it follows that $V_T(\omega) + \phi_T - \beta^{-1}\, g(e_{T-1}) + g(\hat{e}_T(\omega)) \geq V_T^D(\omega)$.

Finally, for marginal dollar of equity issued to be a non-negative value transaction, we must have $g'(e_T(\omega)) \geq 0$ and $g'(e_{T-1}) \geq 0$. This implies that $g'(\hat{e}_T(\omega)) \geq 0$ as well, a result we use below.

These observations together imply that the value of equity at time $T-1$ under $\hat{\mathcal{C}}$ is given by

$$V^E_{T-1}(\hat{\mathcal{C}}) \;=\; V_{T-1} + \phi_{T-1} - V^D_{T-1}(\mathcal{C}) - L_{T-1}(\mathcal{C}) - \beta\, E_{T-1}[m(\hat{e}_T(\omega))]. \tag{A.22}$$

If we can show that $m(\hat{e}_T(\omega)) \leq m(e_T(\omega)) + \beta^{-1}\, m(e_{T-1})$ then it follows from equations (A.20) and (A.22) that $V^E_{T-1}(\hat{\mathcal{C}}) \geq V^E_{T-1}(\mathcal{C})$, whence we will have established that a policy of not issuing equity at time $T-1$ is always optimal. But this is straightforward. By concavity of $m(\cdot)$,

$$\begin{aligned} m(e_T(\omega)) + \beta^{-1}\, m(e_{T-1}) \;&\geq\; (1+\beta^{-1}) \cdot m\left(\frac{1}{1+\beta^{-1}}\, e(\omega) + \frac{\beta^{-1}}{1+\beta^{-1}}\, e_{T-1}\right) \\ &\geq\; m\left(e_T(\omega) + \beta^{-1}\, e_{T-1}\right) \end{aligned}$$

Adding $e_T(\omega) + e_{T-1}$ to both sides, we obtain

$$g(\hat{e}_T(\omega)) \;=\; g(e(\omega)) + \beta^{-1}\, g(e_{T-1}) \;\leq\; g\left(e_T(\omega) + \beta^{-1}\, e_{T-1}\right)$$

Then, since $g'(e_T(\omega)) > 0$, it follows that $\hat{e}_T(\omega) \leq e_T(\omega) + \beta^{-1}\, e_{T-1}$. This combined with definition of $\hat{e}_T(\omega)$ implies that $m(\hat{e}_T(\omega)) \leq m(e_T(\omega)) + \beta^{-1}\, m(e_{T-1})$.

Period $t < T-1$ Since it is optimal not to issue any equity at time $T-1$, the problem of issuing equity at time $T-2$ is exactly identical to the one at $T-1$ considered above, with ω being the representative state possible at time T conditional on a current state V_{T-2}. The argument thus extends to any $t < T$, completing the proof. ☐

A.3.2 The Case of Convex $m(\cdot)$

Intuitively, when $m(\cdot)$ is convex, the cost of issuance can be reduced by doing multiple equity issuances. The following is a specific instance where interim equity issuance is optimal.

Example A.2 Consider the parameterization af Example A.1 with the only change that equity issuance costs are quadratic: $m(e) = 0.1\, e^2$. It is easily checked that the maximum amount of cash that can be raised from new equity issuance is 2.5 (corresponding to $e = 5$).

Suppose first that no equity is issued in period-0 but equity is raised optimally at period 1, if required. Then, the maximum cash available as reserves in period 1 is, as earlier, $\phi^{max} = 5.263$. Also, as demonstrated in Example A.1, strategic debt-service is feasible in state d if $\phi \geq 1$, but even with $\phi = \phi^{max}$, strategic debt-service is infeasible in state u *unless additional equity is issued*. We explore this option next.

The amount of cash required in state u to perform strategic debt-service is $D^L_T(uV_0, \phi) = \min[15, 15+0.5\,\phi] = 15$. The available cash flows to the firm are: (i) new cash flow $uf_0 = 5.5$,

(ii) cash carried forward ϕ, and (iii) new cash raised through equity issuance $e - m(e)$. Since $\phi^{max} = 5.263$ and the maximum cash that can be raised through equity is 2.5, strategic debt-service is infeasible in this state. It follows that the optimal cash management policy is $\phi^* = 1$ as in Example A.1, resulting in time–0 equity and debt values of

$$V_0^E \;=\; 44.90 \;\text{ and }\; V_0^D \;=\; 9.7375$$

Consider instead the following strategy for the firm: (i) issue $e_0 = 5$ units of equity in period 0 raising net amount of $e_0 - m(e_0) = 2.5$, so that $\phi^{max} = (5 + 2.5)(1 + \rho) = 7.895$; (ii) carry $\phi = \phi^{max}$ forward to period-1; (iii) raise an additional amount of $15 - 5.5 - 7.895 = 1.605$ in state u by issuing additional equity $e_1 = 2.009$ in this state (this will net $e_1 - m(e_1) = 1.605$). Now there is no liquidation in state u, and the continuation values are

$$V_1^E(uV_0, \phi) \;=\; 102.491 \qquad V_1^D(uV_0, \phi) \;=\; 15$$

Since $\phi > 1$, strategic debt-service can be performed and liquidation avoided state d too; equity and debt values in this state are

$$V_1^E(dV_0, \phi) \;=\; 88.947 \qquad V_1^D(dV_0, \phi) \;=\; 8.947$$

It follows that under this strategy, initial equity and debt values are

$$V_0^E \;=\; \beta\left[pV_T^E(uV_0, \phi) + (1-p)V_T^E(dV_0, \phi)\right] - m(e_0) \;=\; 85.93$$

$$V_0^D \;=\; \beta\left[pV_T^D(uV_0, \phi) + (1-p)V_T^D(dV_0, \phi)\right] \;=\; 11.375$$

Thus, interim equity issuance strictly dominates not issuing new equity till maturity. □

A.4 Comparison with Fan and Sundaresan [14]

Fan and Sundaresan [14] develop a model with optimal cash management and strategic debt service. However, their model does not allow for costly equity issuance, and the nature of retained cash is different to ours. In our model, retained cash is held as reserves; in Fan and Sundaresan, retained cash is invested in the firm thereby changing the scale of the firm. The two assumptions have different implications. Here is a simple example that shows that debt and equity values could be lower under the Fan-Sundaresan model than ours:

Example A.3 Consider the two–period binomial cash flow model detailed in Example A.1. To incorporate Fan and Sundaresan–type reinvestment in the firm, assume that if an amount s out of firm's available cash is reinvested in the firm, then the firm's scale increases to $V_0 + s$.

Thus, for example, if the state u occurs next period, the value of current and future cash flows viewed from that point on will be $u(V_0 + s)$. Thus, for the specific parameter values of the example, one-period ahead cash-flow values are

$$V_u \;= 1.1\,(100 + s) \;=\; 110 + 1.1\,s \qquad V_d \;= 0.9\,(100 + s) \;=\; 90 + 0.9\,s.$$

As a consequence, the post–liquidation values of the firm in these states are

$$u\,(V_0 + s) - l_0 - l_1 \cdot u\,(V_0 + s) \;=\; 15 + 0.55\,s$$

$$d\,(V_0 + s) - l_0 - l_1 \cdot d\,(V_0 + s) \;=\; 5 + 0.45\,s$$

Note that $s \leq s^{max} = f_0 = 5$. Thus, the cash required to perform strategic debt service in the states is

$$D_T^L(u, s) \;= \min\,[15, 15 + 0.55s] \;=\; 15 \;\; D_T^L(d, s) \;= \min\,[15, 5 + 0.45s] \;=\; 5 + 0.45\,s$$

However, the cash availability in the two states is

$$f_u(s) \;=\; b\,V_u(V_0, s) \;=\; 0.05\,(110 + 1.1s) \;=\; 5.5 + 0.055\,s$$

$$f_d(s) \;=\; b\,V_d(V_0, s) \;=\; 0.05\,(90 + 0.9s) \;=\; 4.5 + 0.045\,s$$

It can be verified that $f_u(s) < D_T^L(u, s)$ and $f_d(s) < D_T^L(d, s)$ for all $s \in [0, s^{max}]$. Hence, liquidation cannot be avoided in either state, so the optimal reinvestment policy is $s^* = 0$ (which is also the residual policy). The time–0 values of equity and debt are

$$V_0^E \;=\; 5 \qquad V_0^D \;=\; 9.5$$

On the other hand, we have shown that in our cash reserves setting, the optimal policy is to set $\phi^* = 1$, resulting in equity and debt values of

$$V_0^E \;=\; 44.90 \qquad V_0^D \;=\; 9.7375.$$

□

The converse situation can also arise. In the Fan and Sundaresan model, reinvestments in the firm compound over time; over several periods, the cash flows of the scaled firm may exceed the cash flows under a cash reserves assumption such as ours. In this case, their model would overstate equity and debt values relative to ours. Put differently, the relation between the models is specific to the parameterization under consideration.

Of course, in the final analysis, the approaches are complementary rather than competitive. Fan and Sundaresan capture the notion of firm growth over time, while we capture the empirically-relevant notion of cash "reservoirs" held as insurance.

Table 1: The Valuation Impact of Cash Management Policy and Equity Issuance Costs

This table describes equity and debt values (V^E and V^D, respectively) and spreads of debt yields over the risk-free rate under residual and optimal dividend policies for the binomial cash flow model when debt-service is strategic. The initial firm value is normalized throughout to $V_0 = 100$, and the zero-coupon debt is taken to have a maturity of 10 years. Each period of the binomial tree is taken to be of 6 months or 0.5 year. The constant b that relates V_t to f_t is fixed at 0.025. The interest rate ρ in each period is set to 0.0247 (i.e., the discount rate $\beta = 0.976$) which when compounded yields an annual rate of 5%. The annualized volatility of the $\{V_t\}$ process is set to $\sigma^2 = 0.10$ (this corresponds to an annualized volatility of about $\sigma = 31.6\%$). The sizes of the up and down moves and the probability of the up move, denoted as u, d, and p, respectively, are calibrated to match this annualized volatility subject to the constraints $u = 1/d$ and $b = [1 - \beta(pu + (1-p)d)]$. The remaining parameters are as described in the table. $\bar{c}$ is the face-value of zero-coupon debt, m_0 and m_1 are the fixed and proportional costs of raising new equity, respectively, and l_0 and l_1 are the fixed and proportional costs of liquidation, respectively. m_0 is taken to be zero. The columns under the heading "Difference" present the difference between residual and optimal dividend policies for V^D, V^E, and corporate bond spread over the risk-free rate. All spreads and spread differences are stated in basis points.

$l_0 = 0.0, l_1 = 0.25$										
		Residual Dividend Policy			Optimal Dividend Policy			Difference		
$\bar{c}$	m_1	V^E	V^D	Spread	V^E	V^D	Spread	V^E	V^D	Spread
0.25	0.00	86.8	13.2	128.3	86.8	13.2	128.3	0.00	0.00	0.00
	0.15	84.7	13.2	128.3	86.4	13.2	125.5	1.63	0.04	2.86
	0.99	72.7	13.2	128.3	86.1	13.7	93.0	13.31	0.47	35.32
0.50	0.00	78.6	21.4	341.3	78.6	21.4	341.3	0.00	0.00	0.00
	0.15	75.1	21.4	341.3	76.9	22.0	312.9	1.87	0.60	28.42
	0.99	64.0	21.4	341.3	75.7	21.9	316.2	11.69	0.53	25.13
0.75	0.00	73.2	26.8	530.1	73.2	26.8	530.1	0.00	0.00	0.00
	0.15	68.7	26.8	530.1	70.5	27.4	505.8	1.79	0.63	24.33
	0.99	58.6	26.8	530.1	67.5	27.2	515.4	8.97	0.38	14.68

$l_0 = 0.2, l_1 = 0.25$										
		Residual Dividend Policy			Optimal Dividend Policy			Difference		
$\bar{c}$	m_1	V^E	V^D	Spread	V^E	V^D	Spread	V^E	V^D	Spread
0.25	0.00	90.9	9.1	506.5	90.9	9.1	506.5	0.00	0.00	0.00
	0.15	89.5	9.1	506.5	90.3	9.1	506.5	0.81	0.00	0.00
	0.99	68.0	9.1	506.5	88.5	11.5	270.8	20.56	2.33	235.67
0.50	0.00	85.0	15.0	714.6	85.0	15.0	714.6	0.00	0.00	0.00
	0.15	82.6	15.0	714.6	83.8	15.5	678.5	1.20	0.52	36.17
	0.99	61.6	15.0	714.6	78.2	19.0	467.0	16.67	3.96	247.63
0.75	0.00	80.7	19.3	878.2	80.7	19.3	878.2	0.00	0.00	0.00
	0.15	77.5	19.3	878.2	79.1	19.4	876.0	1.57	0.04	2.20
	0.99	57.2	19.3	878.2	69.6	22.0	741.2	12.42	2.62	137.06

Table 2: Residual Dividend Policy without Strategic Debt service

This table describes the difference in equity and debt values (V^E and V^D, respectively) and spreads of debt yields over the risk-free rate under residual dividend policies for the binomial cash flow model when debt service is strategic and non-strategic. The equity and debt values and spread of debt yield over the risk-free rate for the non-strategic case are contained in Columns 3, 4 and 5, respectively. The corresponding values for the strategic case are from Columns 3, 4 and 5, respectively, of Table 1. The parameter combinations are the same as in Table 1. The remaining parameters are as described in the table. $\bar{c}$ is the face-value of zero-coupon debt, m_0 and m_1 are the fixed and proportional costs of raising new equity, respectively, and l_0 and l_1 are the fixed and proportional costs of liquidation, respectively. m_0 is taken to be zero.

$l_0 = 0.0, l_1 = 0.25$							
$\bar{c}$	m_1	V^E	V^D	Spread (bps)	Diff. in V^E	Diff. in V^D	Diff. in Spread (bps)
0.25	0.00	86.2	13.2	123.7	0.57	-0.06	4.61
	0.15	84.4	13.2	123.7	0.32	-0.06	4.61
	0.99	44.3	13.2	128.3	28.44	0.00	0.00
0.50	0.00	76.1	22.6	284.2	2.47	-1.22	57.17
	0.15	73.8	21.4	341.3	1.25	0.00	0.00
	0.99	41.9	21.4	341.3	22.07	0.00	0.00
0.75	0.00	69.2	28.3	473.6	4.04	-1.48	56.51
	0.15	66.5	26.8	530.1	2.21	0.00	0.00
	0.99	41.4	26.8	530.1	17.13	0.00	0.00

$l_0 = 0.2, l_1 = 0.25$							
$\bar{c}$	m_1	V^E	V^D	Spread (bps)	Diff. in V^E	Diff. in V^D	Diff. in Spread (bps)
0.25	0.00	88.2	11.8	244.9	2.62	-2.62	261.54
	0.15	86.4	11.8	244.9	3.08	-2.62	261.54
	0.99	46.3	9.2	500.6	21.67	-0.05	5.87
0.50	0.00	78.1	19.3	446.9	6.87	-4.33	267.74
	0.15	75.8	16.0	643.0	6.78	-1.04	71.67
	0.99	43.9	15.1	711.0	17.63	-0.05	3.64
0.75	0.00	71.2	22.9	695.5	9.50	-3.58	182.73
	0.15	68.5	19.5	870.8	8.97	-0.13	7.48
	0.99	43.4	19.4	875.4	13.74	-0.05	2.87

Table 3: Optimal Dividend Policy without Strategic Debt service

This table describes the difference in debt and equity values (V^E and V^D, respectively) and spreads of debt yields over the risk-free rate under optimal dividend policies for the binomial cash flow model when debt service is strategic and non-strategic. The equity and debt values and spread of debt yield over the risk-free rate for the non-strategic case are contained in Columns 3, 4 and 5, respectively. The corresponding values for the strategic case are from Columns 6, 7 and 8, respectively, of Table 1. The parameter combinations are the same as in Table 1. The remaining parameters are as described in the table. $\bar{c}$ is the face-value of zero-coupon debt, m_0 and m_1 are the fixed and proportional costs of raising new equity, respectively, and l_0 and l_1 are the fixed and proportional costs of liquidation, respectively. m_0 is taken to be zero.

$l_0 = 0.0, l_1 = 0.25$							
$\bar{c}$	m_1	V^E	V^D	Spread (bps)	Diff. in V^E	Diff. in V^D	Diff. in Spread (bps)
0.25	0.00	86.2	13.2	123.7	0.57	-0.06	4.61
	0.15	86.1	13.2	123.7	0.28	-0.02	1.75
	0.99	86.1	13.7	93.0	0.00	0.00	0.00
0.50	0.00	76.1	22.6	284.2	2.47	-1.22	57.17
	0.15	76.0	22.1	308.2	0.90	-0.10	4.67
	0.99	75.4	21.9	316.1	0.28	-0.00	0.06
0.75	0.00	69.2	28.3	473.6	4.04	-1.48	56.51
	0.15	68.8	27.6	500.1	1.73	-0.15	5.69
	0.99	66.4	27.2	515.4	1.10	-0.00	0.07

$l_0 = 0.2, l_1 = 0.25$							
$\bar{c}$	m_1	V^E	V^D	Spread (bps)	Diff. in V^E	Diff. in V^D	Diff. in Spread (bps)
0.25	0.00	88.2	11.8	244.9	2.62	-2.62	261.54
	0.15	88.1	11.8	244.9	2.22	-2.62	261.54
	0.99	88.0	11.0	311.2	0.52	0.44	-40.45
0.50	0.00	79.1	19.9	415.5	5.89	-4.93	299.15
	0.15	78.8	19.9	415.5	5.00	-4.41	262.98
	0.99	77.8	18.4	499.3	0.46	0.58	-32.33
0.75	0.00	71.5	24.0	647.4	9.15	-4.64	230.89
	0.15	71.2	22.3	722.9	7.92	-2.96	153.11
	0.99	68.8	21.4	769.7	0.81	0.58	-28.55

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