The impact of risk regulation on price dynamics[*]

Jón Daníelsson   Hyun Song Shin
Jean-Pierre Zigrand
Financial Markets Group, London School of Economics

Revised, January 2003

Abstract

Most financial risk regulations assume that asset returns are exogenous, where risk is estimated from historical data. This assumption fails to take into account the feedback effect of trading decisions on prices. We investigate the consequences of risk constrained trading by means of simulations of a general equilibrium model with a value-at-risk constraint and compare the results to the case when risk constraints are not present. Prices are lower on average in the presence of risk regulation, while volatility is higher. Risk regulation may have the perverse effect of exacerbating price fluctuations.

1   Introduction

The adoption of risk-management techniques by fund managers and other market participants has proceeded at a rapid pace in recent years. Their adoption has been driven not only by the market participants themselves in reaction to their experiences of market turbulence, but also by a regulatory climate that encourages putting so-called *market sensitive* risk management systems at the operational center of financial institutions. The 1996 amendment of the Basel Accord on regulatory capital for market risk (Basel Committee, 1996) has been especially influential in this development, and the central theme of risk based regulation has been developed further to include the balance sheets of banks in the current round of discussions under the Basel II framework.

The imposition of market sensitive regulatory capital stems from the prudential objective of limiting the risk of failure of an individual institution by providing an appropriate buffer against external shocks. In most instances, the existence of such a buffer would be consistent with the self-interested decisions of the financial institution itself. However, the need for the imposition of regulatory capital arises when the incentives of the individual market participant point toward socially sub-optimal risk-taking due to agency problems within the organization. The case for regulatory capital is reinforced by the consumer protection mandate of the regulator where the need is to protect vulnerable uninformed stakeholders of the institution such as small depositors of a bank.

However, there have been well-aired concerns that the widespread adoption of market sensitive risk management practices may have the unintended and undesirable side-effect of exacerbating short term price fluctuations in financial markets. A typical statement of this concern is summarized in the following passage[1].

> "So-called value-at-risk models (VaR) blend science and art. They estimate how much a portfolio could lose in a single bad day. If that amount gets too large, the VaR model signals that the bank should sell. The trouble is that lots of banks have similar investments and similar VAR models. In periods when markets everywhere decline, the models can tell everybody to sell the same things at the same time, making market conditions much worse. In effect, they can, and often do, create a vicious feedback loop."

[1]The Economist Magazine, Oct 12th 2000.

All risk-management systems in widespread use today rest on techniques that attempt to infer the statistical relations governing asset returns by reference to actual, historical realizations of returns. Indeed, this is dictated by the 1996 Basel amendment. By their nature, such risk-management systems treat the uncertainty governing asset returns as being exogenous. They neglect the fact that the behavior of market participants is affected by the adoption of these techniques, creating a feedback effect on the whole financial system. In short, these systems abstract from the system-wide consequences of widespread adoption of these techniques. Blum (1999), Basak and Shapiro (2001), Daníelsson (2002), Daníelsson and Zigrand (2002), Morris and Shin (1999), Crockett (2000), and Persaud (2000) are some recent comments that express unease about the current practice.

To be sure, we are not the first to cite the possible feedback effects of risk-based trading strategies and risk-based regulation. The long list of citations above is but a small subset of the commentary on this issue, both academic and from policy circles. Rather, our incremental contribution lies in two features of our analysis. First, we formalize the feature that risk-based trading strategies must rely on updated forecasts of the underlying statistical relationships, and that in practice such forecasts are based on (recent) historical data. There is, therefore, a disconnect between the true forcing processes that drive the fundamentals and the forecasts of the traders that must grope for the constantly shifting statistical relationships in the market by utilizing historical data. The distinction between the forecasts and the true forcing process is in contrast to existing work on the consequences of VaR constraints such as Basak and Shapiro (2001) where the traders operate under full knowledge of the true forcing process. Our second contribution is to provide a simple, but fully rigorous quantification of the effects of the VaR constraint on market observables. By tracking the evolution of these fundamental variables with and without VaR constraints, we can ask counterfactual questions of a "what if" kind. How would the evolution of prices and forecast volatilities have been different if the VaR constraint were not present, or were of a different threshold? These questions are addressed by means of numerical simulations of our analytical model.

The main themes examined in our paper are eerily reminiscent of the debate in the 1970s arising from the Lucas Critique, (see Lucas, 1976). The main lesson drawn by macroeconomic forecasters then was that one should distinguish between *reduced form regularities* (which are liable to break down when behavior changes) from genuinely *structural relations* of the economy. The failure to distinguish properly between the two lead to unjustifiable conclusions concerning the consequences of policy. This point is succinctly

summarized in *Goodhart's Law*:

> "Any statistical relationship will break down when used for policy purposes" (Goodhart, 1974)

To canny traders who understand the feedback effects in the market, this would be a well-understood point. The fixed income and FX pages of the press and news services abound with references to strategic trading terms such as overhangs of leveraged positions, short–covering and the like. So, it is clear that traders recognize the endogenous nature of market risk, even if their risk management systems do not. The neglect of the endogenous nature of risk in risk management models is all the more puzzling when set against the lessons drawn after the October 1987 crash of the stock market. The Brady Commission (1988) attributed the magnitude and swiftness of the price decline to practices such as portfolio insurance and dynamic hedging techniques. Such trading techniques have the property that they dictate selling an asset when its price falls and buying it when the price rises. Best estimates at the time suggested that around $100 billion in funds were following formal portfolio insurance programs, representing around 3% of the pre–crash market value. However, this is almost certainly an under-estimate of total selling pressure arising from informal hedging techniques such as stop–loss orders, see the survey evidence presented in Shiller (1987). Portfolio insurance has been discussed e.g. by Grossman and Vila (1989), Gennotte and Leland (1990), and Grossman and Zhou (1996). The events of the summer and autumn of 1998 in which financial markets experienced severe disruptions to liquidity and risk appetite can only be satisfactorily understood if the endogeneity of financial risk is taken into account.

To their credit, many regulators have recognized the potentially destabilizing effects of market sensitive risk regulation. In the days following the terrorist attack on New York and Washington on September 11th 2001, financial markets around the world were buffeted by unprecedented turbulence. In response to the short term disruption to the smooth functioning of markets, the authorities responded by suspending various solvency tests applied to large financial institutions such as life insurance firms. In the U.K., for instance, the usual 'resilience test' applied to life insurance companies in which the firm has to demonstrate solvency in the face of a further 25% market decline was suspended for several weeks. More recently, following the renewed declined in stocks markets in the summer of 2002, the Financial Services Authority — the U.K. regulator — has diluted the resilience test so as to pre-empt the destabilizing forced sales of stocks by the major market players.[2]

---

[2]*Financial Times, Weekend Money,* June 28th 2002, "Insolvency rules are eased for

Nevertheless, the fact remains that the overall framework for risk regulation relies on a prudential perspective that views each financial institution on its own. The framework tends to neglect the system-wide perspective. Hence, the blindspot in conventional risk management techniques, and especially the supervisory approach to the regulation of risk, is the presumption that risk management is a single-person decision problem i.e. a *game against nature*. That is, uncertainty governing price movements is assumed to be exogenous, and assumed not to depend on the actions of other decision makers. The analogy is with a meteorologist trying to predict the weather. The weather is unaffected by the predictions issued by weather forecasters and the consequent actions that these forecasts generate. Financial markets are different. When short run price changes are influenced by the trading decisions of market participants (as surely they must), then shifts in the beliefs of market participants will lead to actions that precipitate certain outcomes. There is, in other words, a feedback effect from the beliefs of market participants to the actual outcome in the market. Indeed, there are strong reasons to believe that this feedback effect will reinforce any exogenous pressures on prices arising from the fundamentals.

A gambling analogy is helpful. If the underlying uncertainty facing a trader were exogenous, modelling risk is akin to a gambler facing a spin of a roulette wheel, where the bets placed by him and other gamblers do not affect the outcome of the spin. However, when the outcome depends on the actions of other traders, risk modelling resembles poker more than roulette. Current risk management practices rest on the roulette view of uncertainty. The roulette wheel may have an unknown number of outcomes with differing probabilities, but as long as the outcome is unaffected by the actions of other gamblers, it is simply a matter of applying standard statistical techniques to past outcomes to enumerate what these outcomes are, and to estimate their respective probabilities. Much of the sophisticated techniques in the current state of the art can be seen as alternative ways of refining such estimation procedures, as well as tracking the non–linear payoff structures arising from derivative securities. To the extent that the stochastic process governing asset prices depends on what traders do, this view of the world is invalid. The uncertainty facing traders is endogenous, and depends on the actions of market participants.

In spite of the focus of our paper on possible deterimental effects of risk-sensitive regulation, it is not our intention to argue that such regulation is undesirable. We have not addressed the agency problems or other incentive

---

life offices" by Jason Corcoran.

issues that push individual trader's actions away from some welfare benchmark. Nor have we addressed the consumer protection mandate of the regulator. Both the incentives question and the consumer protection mandate are critical to reach an overall assessment of the desirability of risk-based regulation. The costs of risk regulation must be set against the benefits. Our purpose in this paper is to contribute to a fully informed debate on the issue by highlighting the endogenous nature of risk when market participants are faced with regulatory constraints. The costs of risk regulation must be set against the benefits. Our purpose in this paper is to contribute to a fully informed debate on the issue by highlighting the endogenous nature of risk when market participants are faced with regulatory constraints.

In what follows, we investigate the consequences of the widespread adoption of market-sensitive risk management systems that treat financial market uncertainty as exogenous, and do not take into account the feedback from traders' actions to the market outcomes. We gauge their systemic impact by comparing the dynamics of asset prices with risk–management systems and compare this with asset price dynamics in the absence of such systems. The plan of the paper is as follows. In the next section, we will give a non-technical overview of our model and summarize some of our main simulation results. Section 3 presents the model in more detail, and Section 4 describes our simulations in more detail.

2 Overview of Model and Simulation Results

Our model is based on a standard asset pricing model where a large number of small traders each have constant absolute risk aversion, but where the degree of risk aversion varies across traders. The underlying uncertainty is driven by the cashflow process which is assumed to be conditionally normal. The model builds on the single period model by Daníelsson and Zigrand (2002) in which a Value–at–Risk (VaR) constraint is introduced that restricts the portfolio choices of traders. Specifically, traders forecast next period's risk, and this estimate restricts their portfolio selection. A similar model has been proposed by Basak and Shapiro (2001). Crucially, traders follow current practices in risk management and forecast risk by means of a standard variance–covariance (VCV) technique. In other words, traders use a backward–looking belief revision process in which forecasts are generated by reference to past realizations of returns, as recommended by many of the current risk management systems and required by regulators.

When the VaR constraint binds, we show that it has an effect similar to

an increase in the underlying risk aversion of the traders. Since the degree to which this constraint binds is determined by market outcomes, the net effect of the VaR constraint is that the traders behave *as if* their degree of risk aversion is fluctuating with the market outcomes. In particular, since the VaR constraint binds mostly during periods of market turbulence, the increased *effective risk aversion* leads to sales of risky assets, and serves to exacerbate market volatility and to reduce liquidity. This is indeed one way in which we can understand the frequently heard statement from market pundits during distressed episodes that the "level of risk aversion has gone up." Such statements are normally frowned upon by conventional economists, since risk aversion is part of the make–up of the individual trader (much like the color of his eyes), and are part of the basic parameters of the economy. However, our interpretation suggests that even if the underlying basic preferences remain constant over time, the *actions* of the traders are determined by a rule which is identical to the actions of someone whose risk aversion is fluctuating over time.

The dynamics of the model are generated by the sequence of events where the risk-constrained traders have a set of beliefs from which they base their net asset demands. Market-clearing then generates equilibrium prices. The assets yield exogenous random payoffs which in turn determine realized returns for risky assets. Completing the cycle, the traders use this realization to update their beliefs and the economy advances by one period. The diagram on Page 10 illustrates the sequence of events that generate the dynamics in our model.

We can implement our model once we specify the exogenous stochastic process that generates the payoffs of the assets at the beginning of each date. We use standard linear normal models to generate this exogenous uncertainty, incorporating volatility clustering (GARCH) effects, calibrated to stylized facts about financial data. Our simulations reveal the following effects of the widespread adoption of VaR constraints.

- Prices (for positive–beta assets) are lower on average with VaR constraints than without. The magnitudes can sometimes be very substantial.

- Following a negative shock to asset prices, the troughs in the price paths are deeper, last longer, and take a longer time to recover when there are VaR constraints.

- The forecast variance of returns is, in general, larger with VaR constraints than without. The difference in the forecast variance as be-

tween the constrained and unconstrained economy is largest during distressed episodes in the market.

From our simulations, it seems that the widespread adoption of backward–looking risk management practices, such as standard VCV methods, may have a detrimental effect on asset price volatility and market distress. Far from having a dampening effect on price volatility, it may have the perverse effect of exacerbating it.

3   Model

We now give a more complete description of our model, and describe how a VaR constraint affects traders' actions. Time is discrete, and indexed by $t \in \{0, 1, 2, \cdots\}$. In each period, there are three traded assets, labeled as assets $0, 1$ and 2. Asset 0 is the risk-free asset, while assets 1 and 2 are risky. Without loss of generality we assume there is a fixed, deterministic time-invariant supply of θ_i units of the ith risky asset. We denote by q_{it} the price of the ith asset at time t. Any asset traded at time $t - 1$ matures at date t, and yields payoff d_{it}. The *gross return* on asset i between periods t and $t + 1$ is denoted by $R_{i,t+1}$, and defined as

$$R_{i,t+1} \equiv \frac{d_{i,t+1}}{q_{it}}$$

Denote by $\boldsymbol{R}_t$ the vector of returns over risky assets, so that

$$\boldsymbol{R}_t \equiv \left(\begin{array}{c} R_{1t} \\ R_{2t} \end{array} \right)$$

The traders in our model have short horizons, in the sense that they choose their portfolio to maximize expected utility in the next period. The short horizon assumption tries to capture the incentives that fund managers face in which considerable emphasis is placed on short–term assessments of their performance, or internal controls on their discretion such as 'daily earnings at risk' (DEAR) limits.

To further facilitate the analysis, we reduce the dynamic portfolio choice problem to a sequence of one–period choice problems by assuming that the assets in the economy mature after one period, and traders must roll over their holding in each period. If we denote by x_t^h the number of units of the safe asset held by trader h between periods t and $t + 1$ and by y_{it}^h the number

8

of units of the risky asset i held by trader h between periods t and $t+1$, then the wealth in period $t+1$ for trader h is given by

$$W^h_{t+1} \equiv x^h_t d_{0,t+1} + \sum_i y^h_{it} d_{i,t+1}$$

At date t, trader h aims to maximize the expected value of his von Neumann Morgenstern utility

$$u\left(W^h_{t+1}\right)$$

The traders are also assumed to have constant absolute risk aversion, so that the asset demands are independent of wealth, and prices depend only on the aggregate endowments, rather than the distribution of endowments across traders. In particular, we suppose that there is a continuum of small traders and denote by α^h the coefficient of absolute risk aversion of trader h. The population of traders is such that α^h is distributed uniformly on the interval $[\ell, 1]$ where $\ell > 0$. This last assumption ensures that there are no risk-neutral traders in our model.

Since prices are independent of the distribution of wealth across traders in such a model, we can simplify the notation without affecting the equilibrium prices by assuming that the new supply of the risky assets at each period is owned by individuals other than by the traders themselves. Moreover, we suppose that the aggregate endowments of the risky assets are constant over time. Denote by

$$\boldsymbol{\theta} \equiv \left(\begin{array}{c} \theta_1 \\ \theta_2 \end{array} \right)$$

the vector of time-invariant aggregate supplies of the risky assets.

This model is a multiperiod extension of the Daníelsson and Zigrand (2002) model. Their model analyzes the general equilibrium implications of VaR constraints with emphasis on a formal analysis of the single period problem. Here, we extend the analysis to a multi–period learning framework by means of simulations

3.1 Belief Revision Process

A key feature of our approach is the modelling of the beliefs of the traders. In order to replicate the effect of risk–management systems that rely on realized historical returns, we investigate the effects of a backward–looking belief revision mechanism for traders. We assume that the traders update their beliefs through standard VCV tools for forecasting covariance matrices, in particular, RiskMetrics$^{\text{TM}}$. Furthermore we assume the traders use a similar

forecasting rule for returns. Although it would be straightforward to model updating of beliefs with more sophisticated techniques, such as having the traders optimize a GARCH-type model each period, we believe that this would not add to our results in any significant way.

The traders' beliefs on returns are conditionally jointly normal, and we denote by $(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$ the traders' beliefs concerning the expected returns $\boldsymbol{\mu}_t$ and covariances of returns $\boldsymbol{\Sigma}_t$ of the risky assets between dates t and $t+1$. Thus,

$$\boldsymbol{\mu}_t = \left(\begin{array}{c} \mu_{1t} \\ \mu_{2t} \end{array} \right)$$

$$\boldsymbol{\Sigma}_t = \left(\begin{array}{cc} \sigma_t^{11} & \sigma_t^{12} \\ \sigma_t^{12} & \sigma_t^{22} \end{array} \right)$$

where σ_t^{ij} is the covariance of $R_{i,t+1}$ and $R_{j,t+1}$.

The traders adjust their beliefs concerning the expected returns and covariances by taking a geometric weighted average of past realizations, where the decay factor $\rho = 0.97$ follows the RiskMetrics$^{\text{TM}}$ (1999) recommendations. The backward–looking belief revision mechanism can be formalized in terms of the recursive updating rule given by the mapping

$$((\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t), \boldsymbol{R}_{t+1}) \mapsto (\boldsymbol{\mu}_{t+1}, \boldsymbol{\Sigma}_{t+1})$$

where

$$\sigma_{t+1}^{ij} = \rho \sigma_t^{ij} + (1-\rho)(R_{it} - \mu_{it})(R_{jt} - \mu_{jt}) \tag{1}$$

for constant $\rho \in (0,1)$, and likewise for $\boldsymbol{\mu}_{t+1}$, so that

$$\boldsymbol{\mu}_{t+1} = \rho \boldsymbol{\mu}_t + (1-\rho) \boldsymbol{R}_t \tag{2}$$

The dynamics of our model are generated in the following fashion. The economy begins with an initial set of beliefs for the traders, given by $(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0)$. Based on these beliefs, traders make their portfolio choices. If traders are constrained by a VaR constraint, their portfolio choices will be affected (we will examine the optimal portfolio rules in more detail below). Given the portfolio choices, the aggregate demand functions can be defined. Together with the aggregate endowments $\boldsymbol{\theta}$ we can derive the prices of the assets. Then, the realizations of payoffs $\{\boldsymbol{d}_1\}$ determine the returns $\boldsymbol{R}_1$ for the risky assets, and the traders update their beliefs according to (1) and (2). This is repeated until the simulation ends at $t = T$.
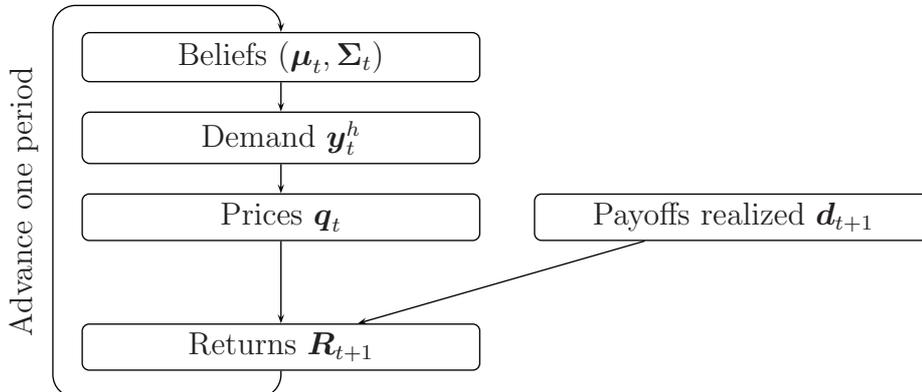
In order to implement our model, we need to solve for the optimal portfolio decisions of the traders, and specify the stochastic process generating the payoffs. We tackle each in turn.

3.2 The Value–at–Risk Constraint

We first solve the portfolio choice problem of the traders, in which they face a possible constraint on the probability of a loss beyond some fixed size. We examine a constraint of the form

$$\text{Prob}_t \left[\text{E}_t \left(W_{t+1}^h \right) - W_{t+1}^h \geq \text{VaR} \right] \leq \bar{p} \tag{3}$$

where VaR is the permitted value-at-risk, and $\bar{p}$ is the permitted probability with which the VaR limit can be breached. Given the conditional normality of beliefs, we can express this constraint in simple terms. Denoting by $\Phi(\cdot)$ the distribution function of the standard normal, note that (3) holds if and only if

$$\Phi \left(\frac{-\text{VaR}}{\sigma_{W,t+1}} \right) \leq \bar{p}$$

where $\sigma_{W,t+1}$ is the standard deviation of W_{t+1}^h with respect to the beliefs at date t. Equivalently, this condition can be written as

$$\sigma_{W,t+1}^2 \leq \left(\frac{\text{VaR}}{\Phi^{-1} (1 - \bar{p})} \right)^2 \equiv \bar{v} \tag{4}$$

Thus, the VaR constraint boils down to a constraint on the maximal permitted variance of W_{t+1}^h. The right hand side of (4) is a constant determined by the parameters of the problem, and we will denote it by $\bar{v}$.

11

It is useful to consider the forecast distribution of payoffs in addition to those for returns. The forecast distribution follows from the updating process (1). Denote by

$$(\boldsymbol{m}_t, \boldsymbol{S}_t)$$

the forecast mean and covariance of payoffs, and denote by

$$s_t^{ij}$$

the (i,j)th entry of $\boldsymbol{S}_t$. Thus, s_t^{ij} is the traders' belief of the covariance between $d_{i,t+1}$ and $d_{j,t+1}$.

We can then incorporate the VaR constraint formally into a trader's portfolio choice problem. Denoting by $\mathrm{E}_t(\cdot)$ the expectations operator with respect to beliefs at date t, trader h's optimization problem at date t is to maximize

$$\mathrm{E}_t\left(u^h\left(W_{t+1}^h\right)\right)$$

by choosing portfolio holdings $x_t^h, y_{1t}^h, y_{2t}^h$, subject to

$$q_{0t}x_t + \sum_i q_{it}y_{it}^h \leq d_{0t}x_{t-1} + \sum_i d_{it}y_{i,t-1}^h$$
$$\sum_i \sum_j s_t^{ij} y_{it}^h y_{jt}^h \leq \bar{v}$$

The first constraint is the period t budget constraint of trader h. The right hand side of the constraint is the available wealth resulting from last period's investment, while the left hand side is the value of the portfolio purchased at time t. The second constraint is the VaR constraint, which can be expressed more succinctly as

$$\boldsymbol{y}_t^{h'} \boldsymbol{S}_t \boldsymbol{y}_t^h \leq \bar{v}$$

The first–order condition for the optimal portfolio can then be written as:

$$\mathrm{E}_t\left[u^{h'}\left(W_{t+1}^h\right)\left(\boldsymbol{d}_{t+1} - d_{0,t+1}\boldsymbol{q}_t\right)\right] = 2\lambda_t^h \boldsymbol{S}_t \boldsymbol{y}_t^h$$

where λ_t^h is the Lagrange multiplier associated with the VaR constraint for trader h at date t. Denote by ϕ_t^h the normalized Lagrange multiplier for the VaR constraint, given by.

$$\phi_t^h \equiv \frac{2\lambda_t^h}{\mathrm{E}_t\left[u^{h'}\right]}$$

Then, the first order condition can be re-arranged to yield the demand functions:

$$\boldsymbol{y}_t^h = \frac{1}{\alpha^h + \phi_t^h} \boldsymbol{S}_t^{-1}\left(\boldsymbol{m}_t - d_{0,t+1}\boldsymbol{q}_t\right)$$

Were it not for the ϕ_t^h term in the denominator, these demand functions are the familiar textbook ones for traders with constant absolute risk aversion. The VaR constraint enters into the problem by raising the effective risk aversion for the trader from α^h to $\alpha^h + \phi_t^h$. In other words, the introduction of the VaR constraint acts in the same way as a fluctuating risk aversion level of the trader.

By setting aggregate demand equal to the supplies $\boldsymbol{\theta}$, we can solve for the equilibrium prices at date t.

$$\boldsymbol{q}_t = \frac{1}{d_{0,t+1}} \left(\boldsymbol{m}_t - \Psi_t \boldsymbol{S}_t \boldsymbol{\theta} \right) \tag{5}$$

where

$$\Psi_t \equiv \frac{1}{\int_\ell^1 \frac{1}{\alpha^h + \phi_t^h} dh}$$

Ψ_t can be given an interpretation in terms of the average "effective risk aversion" across all traders at date t. It is proportional to the harmonic mean of the individual effective risk aversion coefficients $\alpha^h + \phi_t^h$.

The expression for equilibrium prices in (5) suggests that we can make some sense of the frequently heard statement during distressed episodes for the market that the "level of risk aversion has gone up." Our expression for equilibrium prices suggest that even if the basic risk aversion is constant (given by α^h), the trader acts "as if" his risk aversion was fluctuating over time. It is in this sense that we can regard Ψ_t as the average *effective risk aversion* of the traders.

The implementation of our model entails solving for the time series of $\{\Psi_t\}$. There are a number of subtleties in solving for Ψ_t, since the Lagrange multipliers λ_t^h depend on the contemporaneous prices, and hence the solution of Ψ_t involves solving a fixed point problem. The reader is referred to Daníelsson and Zigrand (2002) for the detailed solution method, and for the conditions for the existence of equilibrium. However, the solution for Ψ_t can be given a relatively clean explicit characterization.

In order to state the solution for Ψ_t explicitly, first consider the function ze^z. This is a non-monotonic function that has a minimum at $z = -1$. For $z < -1$, it is a decreasing function for z, while when $z > -1$, it is an increasing function of z. Consider the restriction of this function to the interval $(-\infty, -1]$. The inverse of this restricted function is defined for the interval $[-e^{-1}, 0)$, and takes values in $(-\infty, -1]$. This partial function is sometimes referred to as the non-principal branch of the Lambert correspondence. We denote this partial function as $F(\cdot)$. Then, the explicit solution for Ψ_t can

be stated as follows (see Daníelsson and Zigrand, 2002)

$$\Psi_t = \begin{cases} \dfrac{1}{\ln \ell^{-1}} & \text{if} \quad 0 {\leq} \kappa_t {\leq} \ell \ln \ell^{-1} \\[2ex] \dfrac{\kappa_t - \ell}{\kappa_t F\left(-\left(\kappa_t + \ell\right) e^{-1}\right)} & \text{if} \quad \ell \ln \ell^{-1} {\leq} \kappa_t {\leq} 1 - \ell \end{cases}$$

where

$$\kappa_t \equiv \sqrt{\frac{\boldsymbol{\theta}' \boldsymbol{S}_t \boldsymbol{\theta}}{\bar{v}}}$$

An equilibrium fails to exist when $\kappa_t > 1 - \ell$. The intuition for this lies in the interpretation of κ_t. The numerator is the total risk in the economy that must be borne by the traders as a whole, while the denominator is the maximum allowable risk. The total measure of traders is $1 - \ell$, and equilibrium fails to exist when the per capita risk in the economy is too large for the traders to take on.

4   Simulations

We now come to the core of our paper, where we report the simulation results of our model. The traders in our model do not know the data generating process (DGP), and hence can only indirectly infer the DGP from realized returns. We follow standard practice in risk management, and assume that the traders employ a backward looking updating rule to estimate the latent DGP. In particular, the traders employ a standard variance-covariance (VCV) method, i.e. the RiskMetrics model. Learning in our model therefore mirrors standard practice in the actual market, where in general, the perceived DGP believed by traders, is different from the actual latent DGP. This is in contrast to previous papers in the literature, such as Basak and Shapiro (2001) who examine the case where the traders have full knowledge of the model.

In choosing the DGP, our primary objective is to mirror important stylized facts regarding financial returns, in particular volatility clustering, uncon-ditional non–normality, and the relative size difference between returns and volatility for equities. The actual DGP is a daily mulitvariate GARCH pro-cess where the chosen parameter values were obtained by estimating the GARCH model with observed equity returns. Thus, in the results we re-port below, the true payoffs are conditionally normal and unconditionally non–normal, and allow for volatility clustering.

4.1 Parameter Values

The following parameter values for our model were used in the simulations, where $\hat{\boldsymbol{m}}$ is the mean of the latent payoff process, and $\hat{\boldsymbol{S}}$ is the unconditional covariance matrix of the latent payoff process.

endowments	$\boldsymbol{\theta} = \begin{pmatrix} 1.9 \\ 0.5 \end{pmatrix}$
daily risk free rate	$r_0 = 1.00013$
decay factor	$\rho = 0.97$
lowest risk aversion	$\ell = 0.0011$
unconditional latent payoff distribution	$\hat{\boldsymbol{S}} = \begin{pmatrix} 0.6 & 0.25 \\ 0.25 & 0.4 \end{pmatrix}$ $\hat{\boldsymbol{m}} = \begin{pmatrix} 1.5 \\ 1.2 \end{pmatrix}$

The simulations proceed as follows. We initialize the DGP and the learning model with reasonable, but arbitrary, starting values. The simulation then proceeds sequentially, where each day we generate payoffs leading to returns and belief updating. To ensure that our simulations were not affected by the initial conditions, we allow the economy to adjust for 500 periods in the simulations before we start to record the data. Thus, date 0 in our reported results is the 501st actual simulation realization. One simulation run, i, is then the sequence of payoffs and beliefs over some predefined sampling interval $t = 0, \cdots, T - 1$. Different simulation runs employ the same DGP, but distinct sets of random numbers, generated independently.

The main numerical features of our simulations are presented in a series of figures. We first show the result of the typical run, and then we examine the impact of an exogenous shock to the latent payoff process that is the equivalent of a large shock that may be expected once in ten years. Finally, we report how we can track changes in the effective risk aversion, prices and volatility, perceived as well as actual. All figures are presented at the end of the paper.

4.2 Analysis of a Run

We first analyze a typical run in some detail, the sample length is 250 trading days corresponding to 1 year. Two model specifications are considered, the first without any regulatory risk constraints, and the second with regulatory risk constraints. In both cases the DGP and all other model parameters are the same. This implies that the sequence of asset payoffs in both cases is identical, however, since the presence of the risk constraints potentially alters returns, it implies that the learning models for the DGP will be different in the regulated and unregulated economies. Given the sample economy, we set the risk constraint $\bar{v} = 100$. The price evolution of the price of asset 1 is shown in Figure 1. The upper panel tracks the price of asset 1 over time, where we have also superimposed the price path of the unregulated economy (where the VaR constraint does not bind). As we would expect for a positive beta asset, the price path of the regulated economy on average is below the price path of the unregulated economy. This is a natural consequence of the fact that the VaR constraint enters as an increment to the effective risk aversion of the traders. Increased effective risk aversion implies that prices are lower. The lower panel shows the ratio of the unregulated price to regulated price.

Let us note the main features. Since the regulated economy operates under a higher "effective" risk aversion, one would expect lower prices in the regulated economy. The price differences can be very substantial, ranging from zero to 40%. However, in our simulations, there are a number of points where the price rankings are reversed, where the regulated price is higher than the unregulated price. This can be explained in terms of the differences in *beliefs* that are ruling the two cases. Even though the effective risk aversion is always higher in the regulated case, there are points in the simulations where the forecasts of the traders are more optimistic under the regulated case than under the unregulated case. This explains why at some points in the simulation, the price rankings are reversed for short periods of time. Clearly, if the beliefs were identical in both economies, the prices in the regulated economy would be uniformly lower than the prices in the unregulated economy in each and every case.

Figure 2 tracks elements of the traders' forecast covariance matrix of returns, Σ_t. The upper panel reports the forecast variance for the return of asset 1. A similar picture emerges, the imposition of the VaR constraint raises the expected forecast variance. The lower panel shows this more explicitly by reporting the ratio of the unregulated variance to regulated variance for asset 1. Again, as with prices, the differences can be rather substantial.

16

Taken together, the impact of regulatory risk constraints in our economy affects both prices and volatility, with prices decreasing and volatility increasing. The reason is that the risk constraint, by reducing the holdings of the less risk averse traders, effectively causes a shift in asset holdings from the least risk averse to the most risk averse traders, and the only way that can happen is by lowering prices implying higher volatility, since the payoff process is exogenous.

4.3   A Shock

We now examine the impact of a large exogenous one day shock to the payoff realization of asset 1. The magnitude of the shock is calibrated to the expected worst daily outcome in 10 years of a major market index, i.e., the expected minimum of 2,500 realizations. This amount is -5.3. The VaR constraint was set as $\bar{v} = 1000$, with other model parameters the same as in the previous subsection.

In this case we are interested in the expected outcome, which necessitates running multiple simulation runs with the same DGP and other model parameters. The cross-sectional average of these multiple runs provide an estimation of the expected values, and by employing a large number of runs, we can effectively eliminates almost all of the simulation estimation uncertainty. We use 10^6 independent simulation runs to obtain an accurate estimation of the expected outcomes.

The effect of the shock is rather dramatic, as can be seen in Figure 3. There is a steep fall in the price after the shock, followed by a slow recovery. In particular, for the first few days after the shock the crisis deepens through the endogenous feedback in prices. Note how the price path in the regulated economy follows a steeper decline and reaches a lower trough. The trough is deeper, and lasts longer in the regulated economy than in the unregulated one. The recovery of the price also takes longer. The bottom panel shows the ratio of prices across the two economies. The price ratio follows a rather interesting double-humped shape, in which the maximum difference in the prices follows rather late after the shock. At its maximum, the price ratio is almost 20%.

Figure 4 plots the corresponding changes in the perceived volatilities in the economy in terms of the determinants of the forecast covariance matrices $\{\boldsymbol{S}_t\}$. The regulated economy with the VaR constraint has substantially higher values of $|\boldsymbol{S}_t|$, thereby inducing the traders to take more conservative trading decisions, and exacerbating the dynamics further. The center panel

plots the ratio of the determinants across the two economies. The bottom panel tracks the evolution over time of the average effective risk–aversion, Ψ_t as well as the steady–state effective risk–aversion, i.e. without the shock.

4.4 Tracking Effective Risk Aversion

One of the advantages of having an explicit model for the portfolio decisions of the traders is that it allows us to track the effect of a tighter VaR constraint through the economy. There are two ways of doing this. The first is through changes in the average effective risk aversion Ψ_t. Figures 3 and 4 plot the Ψ_t realizations over time following a shock. The second way to show the effect of tighter VaR constraints is through the effects on prices and forecast variances. In figure 5, we illustrate the impact of the severity of the VaR constraint on prices and perceived volatilities.

The solid line is the ratio of the average prices $(q_1+q_2)/2$ averaged across time. We can see that as $\bar{v}$ becomes smaller (so that the VaR constraint binds more tightly), the prices in the regulated economy fall further. We have already commented on the fact that when $\bar{v}$ becomes too small, equilibrium fails to exist in our model. For the parameters chosen in our simulations, this point is reached when $\bar{v}$ hits 12. To the left of this point, equilibrium does not exist. At the opposite extreme, we can see that as $\bar{v}\to\infty$, the constrained and unconstrained economies converge.

Also depicted in figure 5 is the ratio of determinants for the regulated and unregulated economies, as a function of $\bar{v}$. As we would expect, the differences in forecast volatility becomes very large when $\bar{v}$ is small. As $\bar{v}\to\infty$, the difference disappears.

In figure 6, we plot an investor's effective risk aversion $\alpha^h + \phi_t^h$ as a function of both the VaR constraint $\log\bar{v}$ but also of the population risk-aversion parameter α^h (recall that the population α^h is distributed uniformly on the interval $[\ell, 1]$). The kink in the surface marks the boundary where the VaR constraint "kicks in" (i.e. starts to bind). When a trader has a high value of α^h, then the VaR constraint does not bind, and hence does not affect the economy.

5 Conclusions

The exercise in this paper has uncovered some unsettling side-effects of imposing value–at–risk constraints in an economy where traders follow backward–

18

looking belief revision rules. Far from stabilizing prices, the effect of such constraints is to induce behavior that exacerbates the shocks further. Of course, the rationale for regulatory capital remains valid even if our arguments are taken on board. Our simulations do not consider the agency problems in financial institutions and the associated incentive issues. Nor have we addressed the consumer protection mandate of the regulator. The overall case for risk regulation must be based on a cost-benefit analysis in which the limitations to risk-taking behavior is set against the possible damage done by the endogenously generated risk that arises from risk regulation. Our purpose in this paper is to contribute to a fully informed debate on the issue by highlighting the endogenous nature of risk when market participants are faced with regulatory constraints.

References

Basak, S. and Shapiro, A. (2001). Value–at–risk based risk management: Optimal policies and asset prices. *Review of Financial Studies*, 14:371–405.

Basel Committee (1996). *Overview of the amandment to the capital accord to incorporate market risk*. Basel Committee on Banking Supervision.

Blum, J. M. (1999). Do capital adequacy requirements reduce risks in banking? *Journal of Banking and Finance*, vol. 23 no 5.

Brady Commission (1988). Report of the presidential task force on market mechanisms. Technical report, Government Printing Office,Washington D. C.

Crockett, A. (2000). Marrying the micro- and macro-prudential dimensions of financial stability. The General Manager of the Bank for International Settlements; `http://www.bis.org/review/rr000921b.pdf`.

Daníelsson, J. (2002). The emperor has no clothes: Limits to risk modelling. *Journal of Banking and Finance*, 26(7):1273–1296.

Daníelsson, J. and Zigrand, J.-P. (2002). What happens when you regulate risk? Evidence from a simple equilibrium model. Working paper, LSE, `www.RiskResearch.org`.

Gennotte, G. and Leland, H. (1990). Market liquidity, hedging, and crashes. *American Economic Review*, 00:999–1021.

Goodhart, C. A. E. (1974). Public lecture at the reserve bank of Australia.

Grossman, S. and Vila, J.-L. (1989). Portfolio insurance in complete markets: a note. *Journal of Business*, 62, no. 4:473–476.

Grossman, S. and Zhou, Z. (1996). Equilibrium analysis of portfolio insurance. *Journal of Finance*, 51(4):1379–1403.

Lucas, R. E. (1976). Econometric policy evaluation: A critique. *Journal of Monetary Economics*, 1.2 Supplementary:19–46.

Morris, S. and Shin, H. S. (1999). Risk management with interdependent choice. *Oxford Review of Economic Policy*, Autumn 1999, 52 - 62, reprinted in the Bank of England *Financial Stability Review*, 7, 141 - 150, `http://www.bankofengland.co.uk/fsr/fsr07art5.pdf`.

Persaud, A. (2000). Sending the herd off the cliff edge: the disturbing interaction between herding and market-sensitive risk management practices. Larosière prize winning essay, Institute for International Finance, http://www.erisk.com/news/features/persaud.pdf.

RiskMetrics Group (1999). Riskmetrics technical manual. http://www.riskmetrics.com/research/techdocs/index.cgi.

Shiller, R. (1987). Investor behavior in the October 1987 stock market crash: Survey evidence. Technical report, NBER discussion paper 2446.
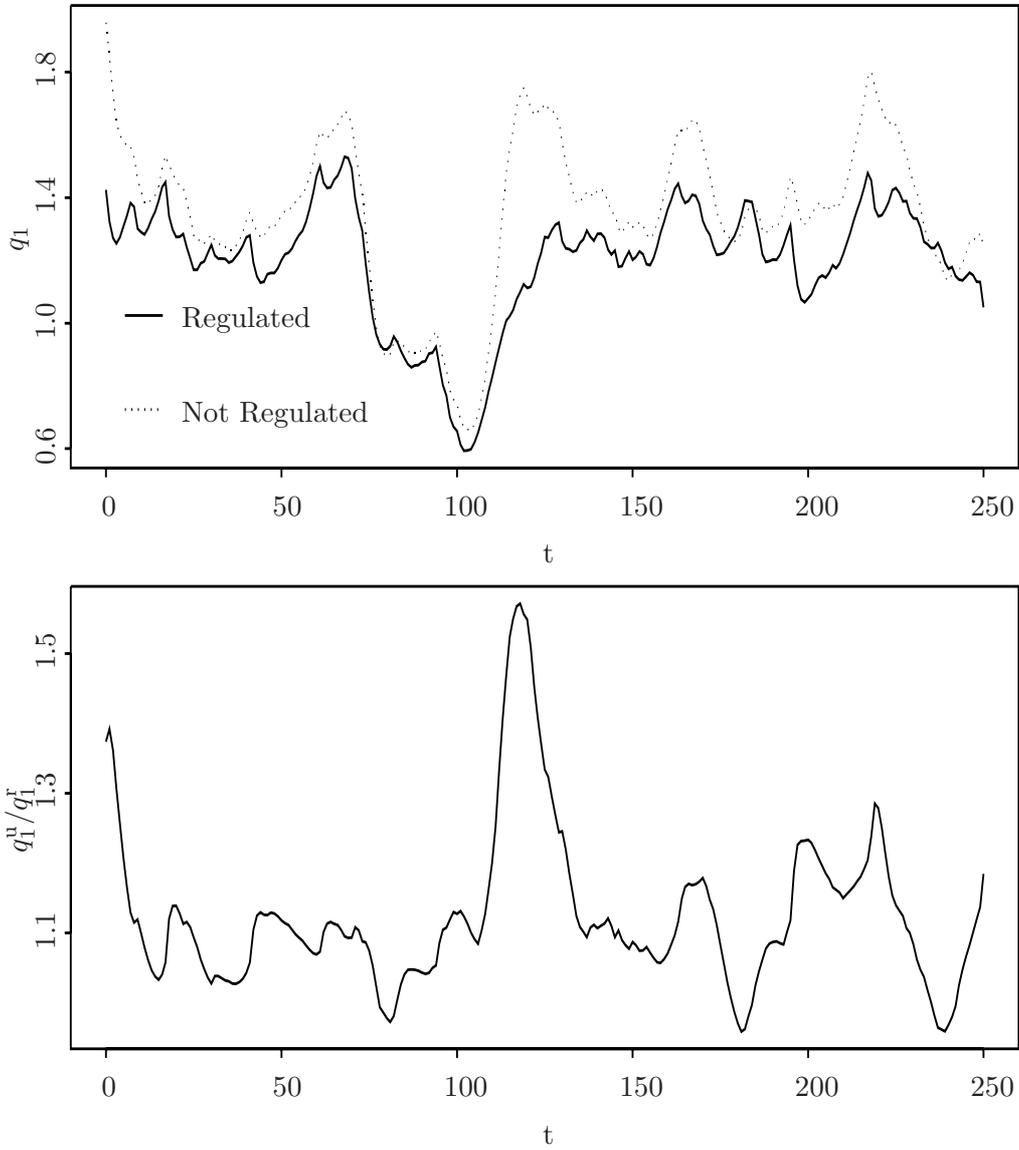
Figure 1: Impact on Prices

250 realizations of the prices of asset one in the sample economy with risk constraint $\bar{v} = 100$. The bottom panel shows the ratio of the regulated to unregulated prices. Note it is greater than one in expectation, indicating that the regulated economy has lower prices on average.
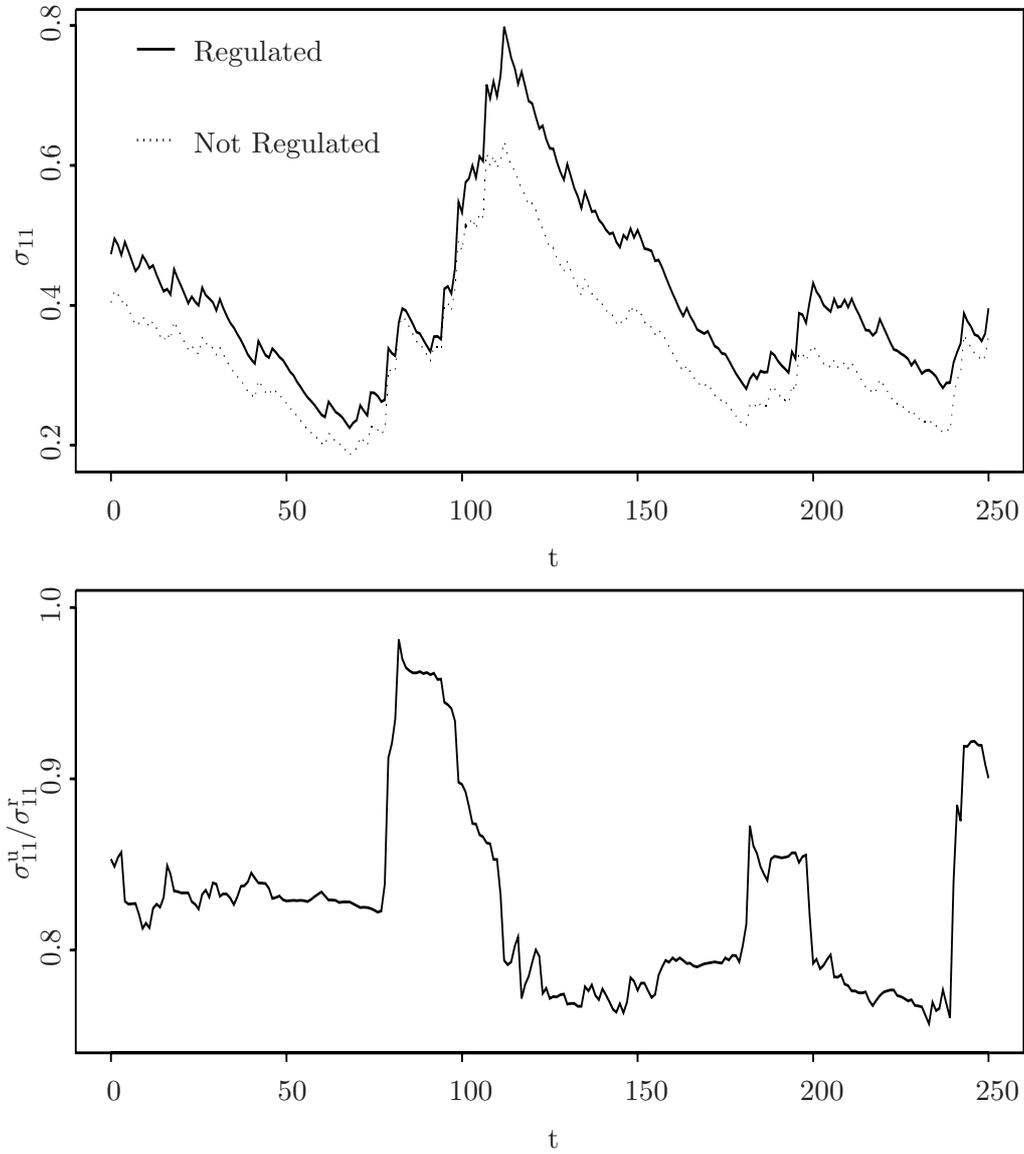
22

Figure 2: Impact on Forecast Return Variance

400 realizations of the forecast volatility of asset one in the sample economy with risk constraint $\bar{v} = 100$. The bottom panel shows the ratio of the regulated to unregulated volatilities. Note it is less than one in expectation, indicating that the regulated economy has higher forecast volatility on average.
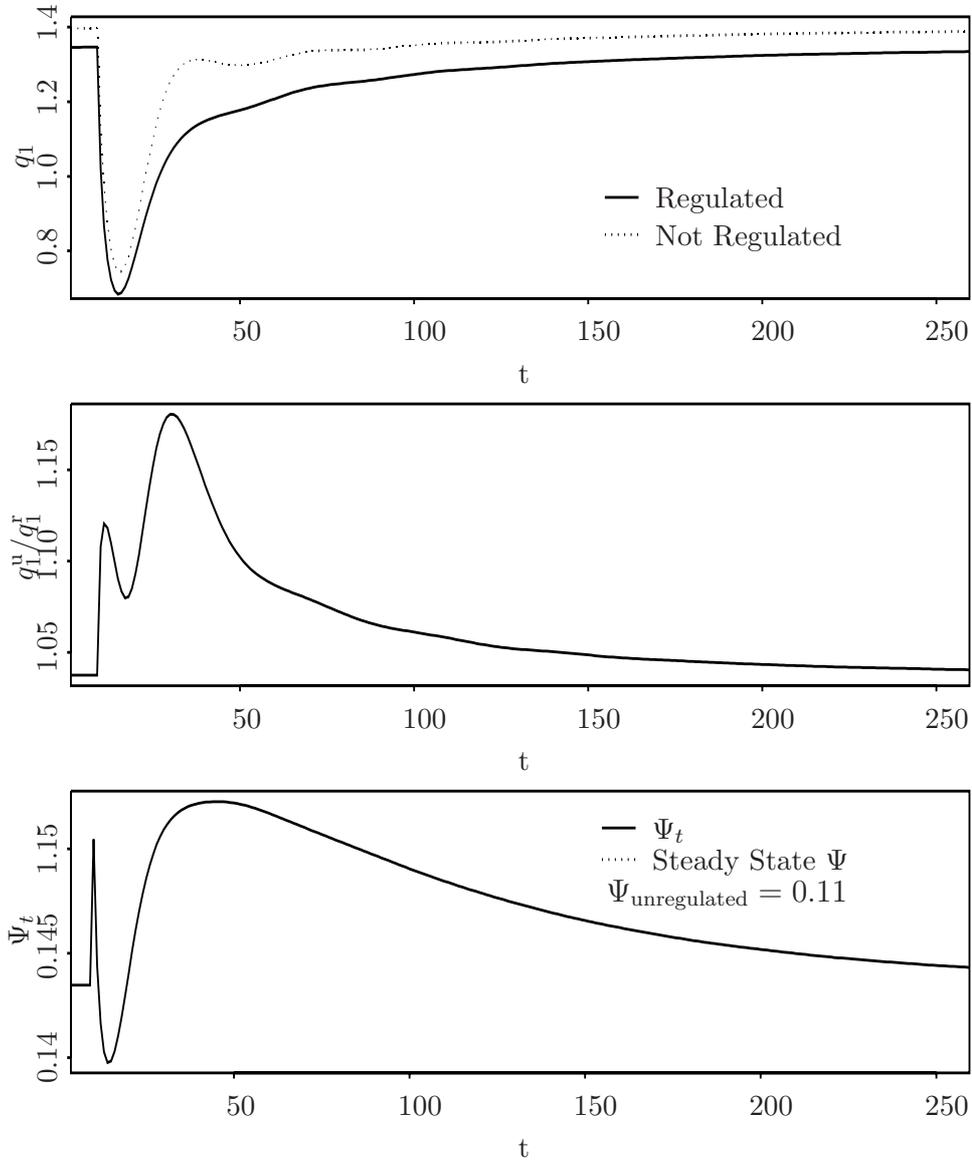
Figure 3: 10 Year Shock to Payoffs: Price Impact

The result of a single period shock equaling the expected largest 10 year (2500 obs.) adverse shock to payoffs of asset one. Economy starts in steady state, and the exercise is repeated 10^6 times. The lower figure shows the ratio of the regulated to unregulated prices. Note that the shock impact is larger, and the adjustment to steady state takes longer in the regulated economy. The bottom panel shows Ψ_t.
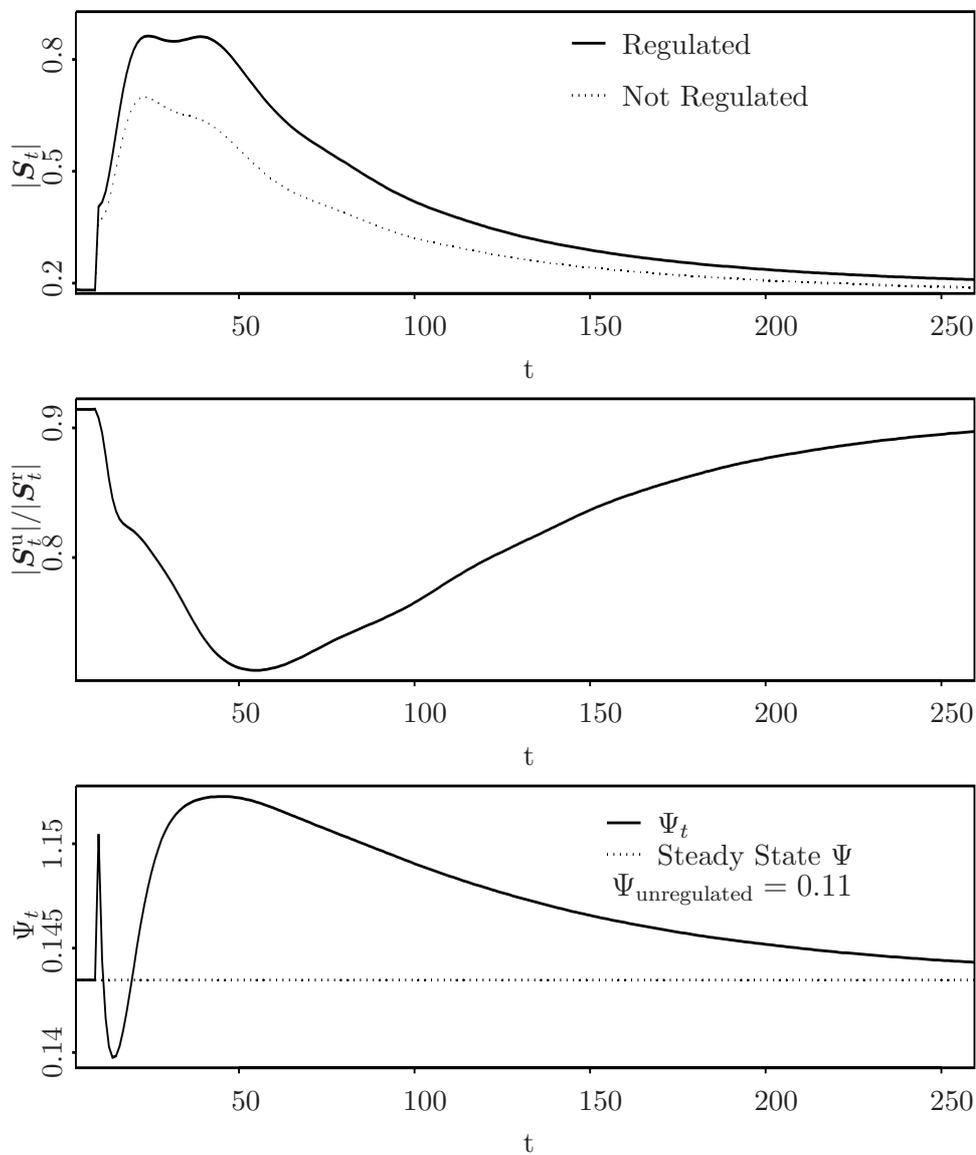
Figure 4: 10 Year Shock to Payoffs: Forecast Volatility Impact

The result of a single period shock equaling the expected largest 10 year (2500 obs.) adverse shock on payoffs on the determinant of the forecast covariance matrix. Economy starts in steady state, and the exercise is repeated 10^6 times. The middle panel shows the ratio of the determinants of unregulated covariance matrix to the regulated covariance matrix. The shock impact is larger, and the adjustment to steady state takes longer in the regulated economy. The bottom panel shows Ψ_t.
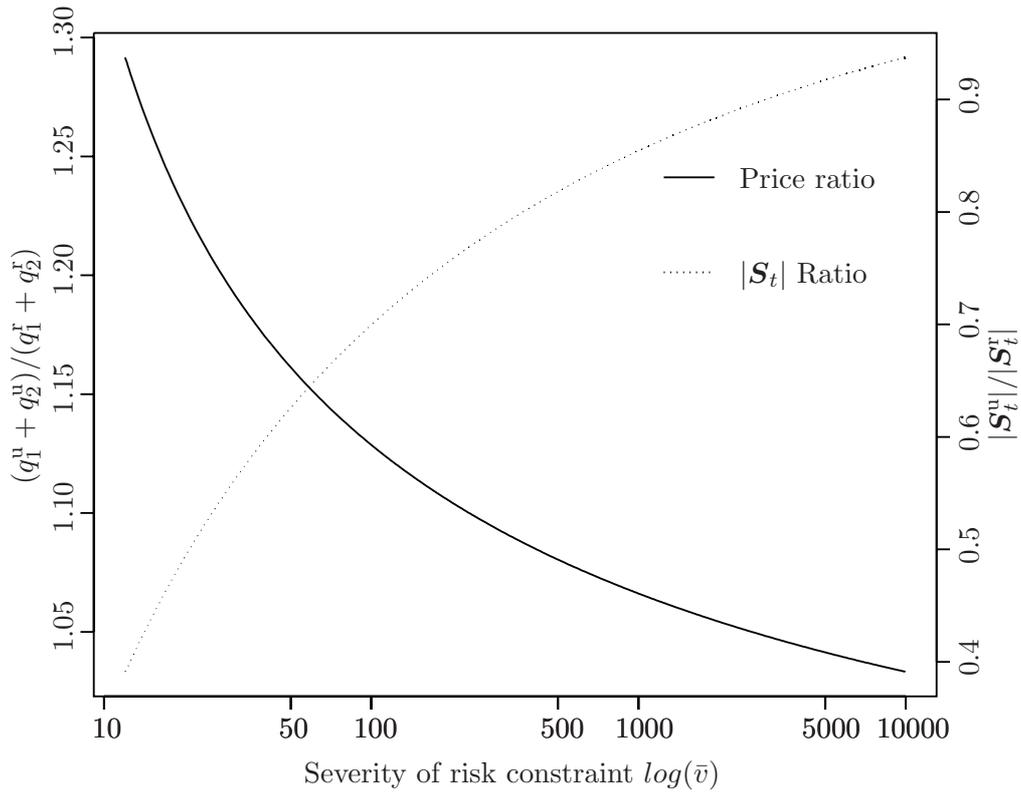
Figure 5: Increasingly Restrictive Risk Constraint

Impact on the sample economy of adjusting the risk constraint, $log(\bar{v})$. At values $log(\bar{v}) <$ 12 equilibrium is no longer defined, and at $log(\bar{v}) = \infty$ the constraint is no longer binding. The figure shows the ratio of the unregulated average prices to the unregulated prices, and the same ratio for the perceived covariance matrix. Note how as $log(\bar{v}) \to \infty$, the ratios tend to one.

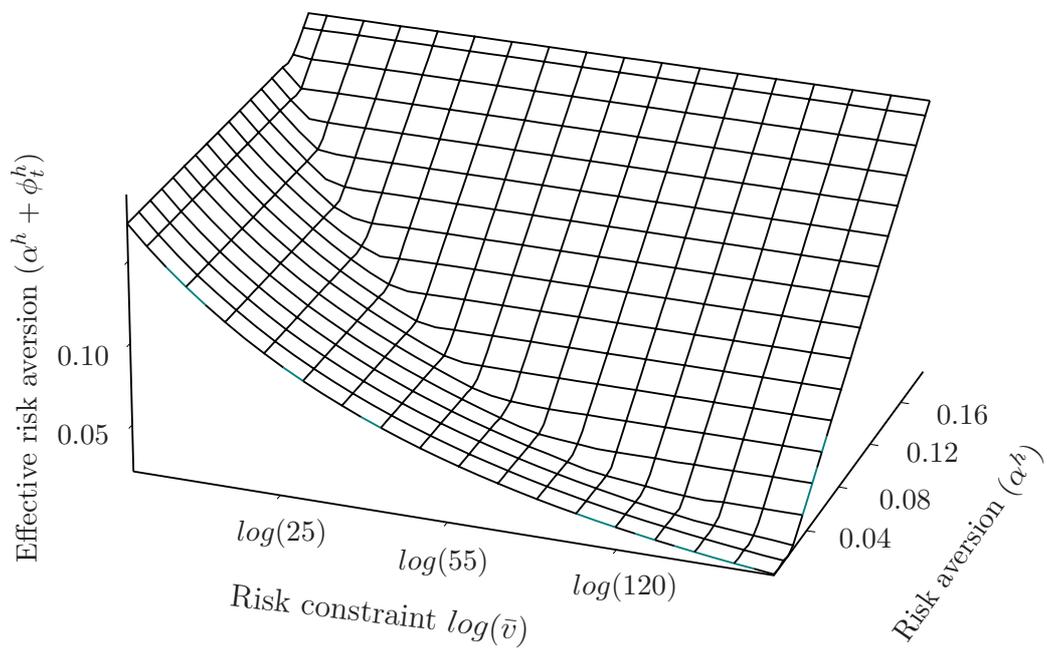Figure 6: Effective Risk Aversion

Impact on effective risk aversion, $(\alpha^h + \phi_t^h)$, of adjusting risk constraint, $log(\bar{v})$, for traders with varying risk aversion, (α^h).