Recursive 'thick' modeling of excess returns and portfolio allocation*

Aiolfi M., C.A.Favero and G.Primiceri
Bocconi University, Bocconi University and CEPR, Princeton University

February 2001

Abstract

This paper explores the extent to which predictability of asset returns could be exploited for dynamic portfolio allocation among several (seven) assets taking model uncertainty explicitly into account.We consider model uncertainty when solving the problem of a representative fund manager who allocates funds between stock and bonds in three geographical areas: Europe, USA and Japan. We consider explicitly model uncertainty by implementing 'thick modelling' to derive the average portfolio allocation generated by the recursively selected top fifty per cent of models in term of adjusted R^2. The portfolio allocation based on this strategy leads to systematic over-performance with respect to optimal portfolio allocation among several assets is based on the predictions of the best model as selected by the adjusted R^2 . Such over performance is mainly attributable to a reduction in the volatility of the returns on the selected portfolios. Thick modelling leads also to systematic replication, but not to over-performance, of a typical benchmark portfolio for our asset allocation problem.

JEL Classification Numbers: G11, C53

*Addres for correspondance: Carlo A. Favero, IGIER-Università Bocconi, Via Salasco 5 20124 Milan Italy e-mail: *carlo.favero@uni-bocconi.it*. We are indebted to Alessandro Penati, Guido Tabellini and Francesco Corielli and seminar participants at 'Ente Einaudi for Monetary and Financial Studies' in Rome for comments and suggestions. All computations have been performed by using MATLAB, codes are available from the authors upon request.

1 Introduction

This paper explores the extent to which predictability of asset returns could be exploited for dynamic portfolio allocation among several (seven) assets taking model uncertainty explicitly into account.

Recent financial research has provided ample evidence on the predictability of asset returns (see, for example, Keim and Stambaugh,(1986), Campbell and Shiller(1988a, 1988b) Pesaran and Timmermann(1996), Lander at al.(1997) and, for a survey, Cochrane(2000)). Such evidence has motivated empirical work on portfolio choice in the presence of time-varying, predictable, expected returns.

The literature typically analyzes portfolio allocation between two assets, stock and cash, and it is divided into two strands. A more theoretical oriented strand (Kandel and Stambaugh,1996, Barberis, 2000) concentrates on very simple predictive models of excess returns to assess the impact of estimation risk, a more empirically oriented strand (Pesaran and Timmermann,1996, 2000) considers richer predictive models to evaluate their portfolio allocation performace against simple buy-and-hold strategies.

Kandel and Stambaugh(1996) evaluate sample evidence about the predictability of monthly stock returns from the perspective of a risk-averse Bayesian investor. They show that the current value of the predictive variables can exert a substantial influence on the portfolio allocation, even when investor's prior beliefs are weighted against predictability.

Barberis(2000) pursues the Bayesian line of research further by concentrating on the estimation risk relevant to models used to predict returns. The empirical results show that there is enough predictability of returns to agree with the often quoted statement by D.Siegel(1994) but they also make clear that ignoring estimation risks might induce a sizeable overallocation to stocks.

Pesaran and Timmermann(1996) consider a richer parameterization for the forecasting model to find that the predictive power of various economic factors over stock returns changes through time and tends to vary with the volatility of returns. They apply a 'recursive modelling' approach according to which at each point in time all the possible forecasting models are estimated and returns are predicted by relying on the best model, chosen on the basis of some given statistical criterion. The dynamic portfolio allocation, based on the signal generated by a time-varying model for asset returns, is shown to over-perform the buy-and-hold strategy. The results obtained for the US are successfully replicated in a recent paper concentrating on the UK evidence, Pesaran and Timmermann(2000).

In this paper we consider model uncertainty when using rich parameterizations for the predictive models. Moreover, we extend the previous empirical work along the dimension of the portfolio allocation by considering the problem of a representative fund manager who allocates funds between stock and bonds in three geographical areas: Europe, USA and Japan.

In the first section of the paper we outline the asset allocation problem and the strategy used to generate the solution. In the second section we describe the data and the specification of the forecasting models. In the third section we propose a dynamic asset allocation, which is evaluated in the fourth section of the paper. The fifth section concludes.

2 The asset allocation problem

We consider the problem of allocating a portfolio among seven assets: we have a safe asset and two asset classes, stock and long-term bonds, for three areas: US, Europe and Japan. The portfolio is rebalanced every month by optimizing the following function:

$$E_t\left(\pi_{t+1}\right) - \frac{c}{2} Var\left(\pi_{t+1}\right) \tag{1}$$

where $E_t\left(\pi_{t+1}\right)$ is the expected return of the portfolio for the following period, $Var\left(\pi_{t+1}\right)$ is the variance of the return of the portfolio for the following period and c is the coefficient of risk aversion. Given that the return for the safe asset in period $t+1$, r_{t+1}, is known at time t, the asset allocation problem is solved by exploiting predictability of asset returns to derive from some estimated model the vector $E_t\left(\mathbf{x}_{t+1}\right)$ of expected values of excess returns for all risky assets, along with the variance-covariance matrix of the one-step ahead forecasting errors $\Sigma_{t+1} = E_t\left(\mathbf{x}_{t+1} - E_t\left(\mathbf{x}_{t+1}\right)\right)\left(\mathbf{x}_{t+1} - E_t\left(\mathbf{x}_{t+1}\right)\right)'$.

The asset allocation problem is then solved by optimizing the following function:

$$
\begin{aligned}
& \max_{\mathbf{a}} \mathbf{a}' \begin{bmatrix} E_t\left(\mathbf{x}_{t+1} - r_{t+1}\right) \\ r_{t+1} \end{bmatrix} - \frac{c}{2}\mathbf{a}'\Sigma_{t+1}\mathbf{a} \\
\mathbf{a}' &= \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & 1 - \sum_{i=1}^{6} a_i \end{bmatrix} \\
\mathbf{x}_{t+1} &= \begin{bmatrix} x_{t+1}^{S,US} \\ x_{t+1}^{S,JP} \\ x_{t+1}^{S,EU} \\ x_{t+1}^{B,US} \\ x_{t+1}^{B,JP} \\ x_{t+1}^{B,EU} \end{bmatrix}
\end{aligned}
$$

where $x_{t+1}^{i,j}$ is monthly return on asset i in area j, with $i = \{S, B\}$ and $j = \{US, JP, EU\}$.

The asset allocation is viewed form the point of view of an investor based in Europe, therefore the safe asset is a 3-month deposit in Euro-DM and all excess returns are denominated in DM. European shares are taken to be German shares prior to January 1999 and truly European shares afterwards, the European bonds are always benchmark German bonds.

The crucial question is the choice of the modelling approach for the prediction of excess returns and the variance-covariance matrix of their forecasting errors.

Importantly, our choice of model focuses on modelling the decision in real-time. Therefore, at any point in time we mimic the decision of an investor who decides portfolio allocation on the basis of the available data.

To this end we implement the recursive modelling approach, according to which at each point in time, t, we search over a base set of observable k regressors to make one-period ahead forecast. In each period we estimate a set of regression spanned by all the possible permutations of the k regressors. As we consider predicting models, simultaneity is not an issue and we estimate our system equation by equation. Moreover the number of regressors k is kept constant for all equations. This gives a total of 2^k different models for each excess return[1]. We keep the sample size constant and all models

[1]Note that in case of simultaneous estimation of all six equations the 'curse of dimensionality' will imply a very rapidly explosive growth of the number of models to be estimated.

run are based on a sample of six years of monthly data. So in each period 6×2^k models are estimated on the previous 72 observations to generate a portfolio allocation. As we keep a fixed window of 72 observations, our methods amounts to running a number of rolling regressions, an alternative could be to proceed to a series of recursive regressions[2], in which case at any point in time the size of the sample used for estimation is increased by one observation.

In practice, we consider monthly data over the period 1984:1-2000:10, our first sample for estimation is 1984:1 -1989:12 which determines portfolio allocation for 1990:1, the data spanning the period 1984:2-1990:1 inform the decision on portfolio allocation for 1990:2 and so on up to 2000:10. Our exercise involves the estimation of $153 \times 6 \times 2^k$ models, which, with k set to 10, amounts to 940032 models.

We estimate all the possible specifications, on an equation by equation basis, of the following system:

$$(\mathbf{x}_{t+1} - r_{t+1}) = \mathbf{Z}_{i,t}\Gamma_{i,t} + \mathbf{u}_{i,t+1} \tag{2}$$

$$\mathbf{x}_{t+1} = \begin{bmatrix} x_{t+1}^{S,US} \\ x_{t+1}^{S,JP} \\ x_{t+1}^{S,EU} \\ x_{t+1}^{B,US} \\ x_{t+1}^{B,JP} \\ x_{t+1}^{B,EU} \end{bmatrix}, \quad \mathbf{u}_{i,t+1} = \begin{bmatrix} u_{i,t+1}^{S,US} \\ u_{i,t+1}^{S,JP} \\ u_{i,t+1}^{S,EU} \\ u_{i,t+1}^{B,US} \\ u_{i,t+1}^{B,JP} \\ u_{i,t+1}^{B,EU} \end{bmatrix}$$

$$\Gamma_{i,t} = \begin{bmatrix} \gamma_{i,t}^{S,US} \\ \gamma_{i,t}^{S,JP} \\ \gamma_{i,t}^{S,EU} \\ \gamma_{i,t}^{B,US} \\ \gamma_{i,t}^{B,JP} \\ \gamma_{i,t}^{B,EU} \end{bmatrix}, \quad \mathbf{Z}_{i,t} = \begin{bmatrix} \mathbf{Z}_{i,t}^{S,US\prime} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{Z}_{i,t}^{S,JP\prime} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{Z}_{i,t}^{S,EU\prime} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{Z}_{i,t}^{B,US\prime} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{Z}_{i,t}^{B,JP\prime} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{Z}_{i,t}^{B,EU\prime} \end{bmatrix}$$

where $\mathbf{Z}_{i,t}^{j,k}$ is the set of regressors, observable at time t, included in the i-th specification ($i = 1, ...2^k$) for the j-th excess return ($j = B, S$) and the k-th

[2] The use Rolling regressions for forecasting allows more parameters'variability over time than recursive regressions .

country$(j = US, JP, EU)$. Given the estimation of (2), the relevant asset allocation problem could, in principle, be solved by specifying a value for c and by setting

$$E_{i,t}\left(\mathbf{x}_{t+1} - r_{t,t+1}\right) = \mathbf{Z}_{i,t}\Gamma_{i,t} \quad (3)$$
$$Var_{i,t}\left(\mathbf{x}_{t+1} - r_{t,t+1}\right) = \mathbf{Z}_{i,t}Var\left(\Gamma_{i,t}\right)\mathbf{Z}'_{i,t} + Var(\mathbf{u}_{i,t+1}) \quad (4)$$

However, while the computation of $E_{i,t}\left(\mathbf{x}_{t+1} - r_{t,t+1}\right)$, can be implemented even if the system is estimated equation by equation, the computation of $Var_{i,t}\left(\mathbf{x}_{t+1} - r_{t,t+1}\right)$ via formula (4) requires simultaneous estimation of the six equations for excess returns which, as already pointed out, runs very rapidly into a classical 'curse of dimensionality' problem. We propose to solve this problem by using the empirical distribution of the forecasting errors. In other words, given the estimation of the model over the first 72 observations, we do not generate portfolio allocation until two years later so that we accumulate at least 24 observations on forecasting errors to compute empirically $Var_{i,t}\left(\mathbf{x}_{t+1} - r_{t,t+1}\right)$.

2.1 Model Uncertainty

Our econometric procedure delivers 2^k models to predict each excess returns and their associated variance-covariance matrices at any point in time, therefore the decision of asset allocation requires us to take a stand on model, or specification, uncertainty.

A traditional approach taken in the literature[3] is to proceed to 'thin' modelling by specifying a selection criteria and therefore by selecting the best model in each period. We follow Granger (2000) and label this approach 'thin' modelling in that the performance of the asset allocation is described over time by a thin line.

Thin modelling needs to be based on a selection criterion which weights goodness of fit against parsimony of the specification. The literature typically considers BIC, AKAIKE, Schwarz and the adjusted R^2 as selection criteria.

The advantage of this approach is that a process potentially non-linear is modeled by applying recursively a selection procedure among linear models. The specification procedure mimics a situation in which variables for predicting returns are chosen in each period from a pool of potentially relevant

[3] See, for example, Pesaran and Timmermann (1995, 2000).

regressors. This choice fits well the behaviour often observed in financial markets of attributing different emphasis to the same variables in different periods.

Obviously keeping track of the selected variables helps the reflection on the economic significance of the 'best' regression.

The main limit of thin modelling is that model, or specification, uncertainty is not considered. In each period the information coming from the discarded $2^k - 1$ is ignored for the portfolio allocation exercise. This choice seems to be particularly strong in the light of the results obtained by Bayesian line of research, which, as we have seen, stresses the importance of the estimation risk for portfolio allocation. A natural way to interpret model uncertainty is to refrain form the assumption of the existence of a "true" model and attach instead probabilities to different possible models. This approach has been labelled 'Bayesian Model Averaging', see, for example, Hoeting J.et al.(1999), and Raftery et al.(1997).

The main difficulty with the application of Bayesian Model Averaging to problems like ours lies with the specification of prior distributions for parameters in all $6{*}2^k$ models of our interest. Recently, Doppelhofer et al.(2000) have proposed an approach labelled 'Bayesian Averaging of Classical Estimates'(BACE) which overcomes the need of specifying priors by combining the averaging of estimates across models, a Bayesian concept, with classical OLS estimation, interpretable in the Bayesian camp as coming from the assumption of diffuse, non-informative, priors.

In practice BACE averages parameters across all models by weigthing them proportionally to the logarithm of the likelihood function corrected for the degrees of freedom, using then a criterion analogous to the Schwarz model selection criterion. It is important to note that the consideration of model uncertainty in our context generates potential for averaging at two different levels: averaging across the different predicted excess returns and their variance covariance matrices and averaging across the different portfolio choices driven by the excess returns and thei variance-covariance matrices. The explicit consideration of estimation risks naturally generates 'thick' modelling, where both the prediction of models and the performance of the portfolio allocations over time driven by those predictions are described by a thick line to take account of the multiplicity of models estimated. The thickness of the line is a direct reflection of the estimation risk. A finding of our empirical work is that the ranking of models in terms of their within sample performance does not match at all the ranking of models in terms of their ex-post portfolio allocation. This empirical evidence points clearly agains BACE using within sample criteria to weight models. Consistently with this evidence, we opted for the selection method proposed by Granger(2000) of using a '... *procedure [which] emphasizes the purpose of the task at hand rather than just using a simple statistical pooling...'*.

We implemented thick modelling by obtaining first the portfolio weights based on the maximization of the CARA utility functions in which mean and variance of the utility function are derived from the best 50 per cent of all our estimated models[4], by considering in turn the portfolio allocation associated with all models for the six relevant excess returns ordered in decreasing order of fit, for a total of $2^k/2$ allocations in each month, and by finally deriving an average portfolio allocation by averaging the optimal weights of each allocation.

3 The data and the econometric specifications

Our application of the recursive modeling strategy requires one-step ahead forecasts for the vector of six excess returns and the associated variance-covariance matrices of the forecasting errors.

Figure 1 displays the monthly excess returns on stock and bond markets for US, Japan and EU[5] over the period 1984:1-2000:10, while Table 1 reports a range of descriptive statistics.

Insert here Figure 1 and Table 1

The monthly excess returns for all variables show very little persistence, their distribution is (moderately) non-normal due to the presence of a number of outliers. Excess returns from stock markets are higher, but more volatile, than those from the bond markets. The higher mean excess returns is that of the US stock market, which takes a value of a 0.0925 on an annual basis. Uncoditional correlations over the full-sample are all positive, however the sub-sample analysis show some instability, which becomes notable for the

[4] The fifty per cent cutoff point has been chosen on the basis of the positivity of the considered criterion

[5] We consider genuine EU data for the stockmarkets and the macroeconomic aggregate from 1999 onwards. Prior to this date German data are considered.

case of the correlation between excess returns of US bonds and stock and European bonds, where the correlation coefficient takes a small negative value in the first sub-sample and an high positive value in the second sub-sample.

To model time-variation in predicted excess returns and their variance-covariance matrices, we apply recursively modelling to obtain time-varying specifications and one-step ahead predictions for these six variables. Following Pesaran and Timmermann(2000) we divide variables in focal, labelled A_t and secondary focal, labelled B_t. Focal variables are always included in all models. We take these variables as those defining the long-run equilibria for the stock and bond markets. Following the lead of traditional analysis[6] (Graham and Dodd Security Analysis, 4th edition, 1962, p.510) and recent studies (Lander et al. (1997)) we have chosen to construct an equilibrium for the stock market by concentrating on a linear relation between the long term interest rates, R_t, and the logarithm of the earning price ratio, *ep*. Such an equilibrium can be derived within the framework of a forward-looking equilibrium models for share-prices (see, for example, Bonfiglioli-Favero(2000)).

For the definition of the long run equilibrium of bond market we use a linear relation between long-term interest rates, R_t and short term interest rates, r_t , which is compatible with the expectational model and is also capable of allowing for a stationary risk premium (see Campbell and Shiller,1987).

Some graphical evidence supporting our choice for the focal variables is provided in figure 2-3 which provides a direct assessment on the performance of our selected focal variables to capture long-run trends in bonds and share prices.

Insert here Figure 2-3

Note that we do not impose any restrictions on the coefficients of the variables entering the long-run equilibria, thus allowing equilibria to change

[6]

> *"...Theoretical analysis suggests that both the dividend yield and the earnings yield on common stocks should be strongly affected by changes in the long-term interest rates. It is assumed that many investors are constantly making a choice between stock and bond purchases; as the yield on bonds advances, they would be expected to demand a correspondingly higher return on stocks, and conversely as bond yields decline..."*

The above statement suggests that either the dividend yield or the earnings yield on common stocks could be used over time. Moreover, in the specification of focal variables for excess returns in the bond markets we consider two lags of the short-term rate to capture both short-run and long-run fluctuations in this variable.

The second set of regressors called 'secondary focal', B_t, includes variables that are always considered important in capturing the short-term effects linked with new data releases or business cycle fluctuations. We consider as secondary focal variables sp_t, the ratio of corporate bonds on government bonds, $sev_t^{i,j}$, a measure of the volatility of asset i in market j[7], $\Delta_{12}lip_t$, the annual rate of change in the index of industrial production, Δlrs_t, the monthly change in the index of retail sales, $\Delta_{12}m3_t$, the annual rate of change of the money supply, $\Delta_{12}cpi_t$, the annual rate of change of retail prices, $\Delta_{12}cpm_t$, the rate of change of commodity price index, $\Delta_{12}oil_t$, the rate of change in the spot price of oil, cc_t, the level of consumer confidence index, bc_t, the level of business confidence index, $demusd_t$, the log of DEM/USD exchange rate, $yendem_t$, the log of YEN/DEM exchange rate, and $yenusd_t$, the log of YEN/USD exchange rate (see the Data Appendix for more details). We initially considered also the lagged dependent variable as a potential semi-focal regressors for all excess returns, but we then discarded it as it was never found significant[8].

Note that some, or even all secondary focal variables, could be left out from the relevant forecasting equation. In fact, given k^a focal variables and k^b secondary focal variables, we construct for all possible samples after iniziatilzation 2^{k^b} models keeping focal variables fixed and considering all the possible combinations of secondary focal variables. We rank then rank models according our proposed criteria and proceed either to thin modelling, choosing only the best model, or to thick modelling, choosing a thick subset of the 2^{k^b} estimated models.

4 Estimation

We estimate, on an equation by equation basis, six one-step(month) ahead forecasting models for the relevant excess returns. The details of these spec-

[7]This measure of volatility is obtained by considering the squared excess returns, it can therefore be interpreted as a measure of conditional volatility.

[8]This empirical results fulfills one of the necessary condition for the generalization to longer term horizons of the optimal short-term portfolio choice (see, for example, Samuleson(1969)).

ifications are reported in Table 2.

Insert Table 2 here

While the number of focal variables is different for excess returns on stock and bonds, we have ten secondary focal variables for each asset and we estimate the same number of specifications for each equation at every sample split.

Figure 4 displays the thick line of the adjusted R^2 associated to the best fifty per cent models (512) for each sample point after initialization. Usually in the literature this criterion is used along with other criteria to offer a wide range of selection of weights in the penalty function which trades off goodness of fit with parsimony. We concentrate just on the R^2 to keep dimensionality under control when evaluating thin against thick modelling. Note that when thick modelling is considered the importance of the selection criterion is drastically reduced. Our sympathy for the adjusted R^2 is generated by the fact that a value of zero for this criterion has an intuitive appeal for the kind of predictive regressions that we run.

Insert Figure 4 here

The value of the criterion ranges from zero to 0.3 and fluctuates rather remarkably over time and so does the thickness of the plot, indicating the presence of differences over time in the relative performance of the estimated models. The start of EMU is associated to a sharp drop in the predictability of excess returns on the stock market but also with a sharp increase in the predictability of excess returns in the bond market.

The importance of recursive modelling is illustrated in Figures 5-10, where we report the probability with which each variable is included in the selected specification, by considering in turn the top fifty per cent models and the top ten per cent model on the basis of the adjusted R^2.

Insert Figures 5-10 here

The plotted probability is computed as the ratio of the number of models in the top ten (fifty) per cent in which the given variable is significant to the total number of models in the top ten (fifty) per cent. The figures emphasize the importance of non-linearities and switching effects for all variables, clearly such importance increases with the thinness of the chosen approach.

5 Asset Allocation

On the basis of the results of estimation we proceed to asset allocation by maximizing the utility function given the forecast for the expected returns and the empirical distribution of the associated forecasting errors. In each period we consider 512 forecasts for each of the six excess returns associated to the best fifty per cent of the estimated models. We then consider 512 portfolio allocations associated with models in decreasing order of adjusted R^2. As we have a total of 129 (total observations used for estimation,153, minus 24 initial observations needed to compute the empirical variance-covariance matrix of forecasting errors) observations on which the optimization exercise is implemented, our exercise involves $129 * 512 = 66\,048$ portfolio allocations. We then proceed to analyze the performance of thin modelling by concentrating on a specific portfolio allocation, usually the portfolio allocation associated to the highest R^2. We can also compare thin modelling with thick modelling based on the average portfolio allocation, derived by averaging the optimal weights of the 512 portfolio allocations.

Figures 11-12 illustrate the optimal weights of the average portfolio allocation by aggregating assets from different areas into bonds and stocks.

Insert Figures 11-12 here

The portfolios chosen by thick modelling tend to favour stocks versus bonds (the average allocation to stock is .62 while the average allocation to bonds is .30) and it implies a rather proactive allocation strategy with frequent and sizeable re-balancing. However,as clearly shown by Figure 12, the volatility of weigths in optimal portfolio chosen by thick modelling is much lower than that generated by thin modelling based on the best R^2.

6 Performance Evaluation

We evaluate the performance of our asset allocation strategies by using a range of tools.

First, we analyze one-step ahead predictions for all six excess returns of our interest by using the sign test proposed by Pesaran and Timmermann(1996). The sign test is based on the proportion of times that the sign of a given variable y_t is correctly predicted in the sample by the sign of the predictor x_t . Under the null hypothesis that x_t has no power in predicting y_t the proportion of times that the sign is correctly predicted has a binomial distribution with known parameters, therefore a test of the null of predictive failure is constructed by comparing the observed proportion of sign correctly predicted with the proportion of sign correctly predicted under the null. Details on the derivation of the statistics and results are reported in Table 3.

Insert Table 3 here

When testing we have considered the whole sample 1990-2000 along with a split in two subsamples, 1990-1995 and 1995-2000. We have analyzed three predictors: the prediction associated with the best model in terms of R^2, the prediction associated with the worst model in terms of R^2, and the average prediction of the top fifty per cent models(512) in terms of R^2. The null of predictive failure is consistently rejected only for excess returns from investing in European shares. The sign test takes positive, but not significant, values for excess returns on European bonds, Japanaese shares and Japanese bonds, while values are consistently negative for the US markets. Evidence from the whole sample dominates evidence form the two subsamples in terms of tendency to reject predictive failure. Interestingly, no clear pattern emerges between the in-sample performance of the selected models as measured by the R^2 and their one-step ahead predictive ability.

The sign tests concentrate on a specific assets of our exercise in that it does not allow to measure performance and it does not consider issues related to the variance-covariance matrix of prediction errors and therefore to the utility function. We then proceed further in our evaluation exercise by analyzing performance. First we look at cumulative performance, by following over time the value of a wealth of 100 in 1992:1. We consider six alternative wealth profiles generated respectively, by the portfolio allocation based on thin modelling with the best R^2, by the portfolio allocation based on thin modelling with the worst R^2, by the portfolio allocation based on thick modelling, i.e. average allocation resulting from the top 50 per cent models in terms of R^2,by the best performance, by the worst performance and by a typical benchmark for funds investing in our asset classes, i.e. an asset allocation with weights 0.55 for bonds and 0.45 for stocks and a country allocation based on GDP weights. The results, reported in Figure 13, show that thick modelling replicates but does not over-performs the benchmark, and that the ranking in terms of R^2 are not reflected at all in the ranking in terms of performance.

Insert Figure 13 here

In fact, the allocation based on best R^2 does not clearly dominate the allocation based on the worst R^2 and the best performance is associated with a model rather low (350) in the R^2 ranking. The evidence from the sign tests and the evaluation of the performance of a rather limited number of models is confirmed when the performance of all 512 models is analyzed: Figure 14 clearly shows that ranking the models in terms of decreasing R^2 does not generate a decreasing performance.

Insert Figure 14 here

This empirical results heavily question the use of within sample performance criteria to proceed to thin modelling and determine the best model for predicting returns and their variance-covariance matrices and determine portfolio allocation. In principle model-selection based on the limited information criteria, such as the adjusted R^2, is dominated by full-information criteria, especially in the presence of several risky assets. The problem of full information criteria is that they are heavily affected by the curse of dimensionality problem (the total number of specifications for our six equation models would be $2^{28} = 268\,435\,456$ as our system includes 28 different semi-focal variables). Moreover, the measure of performance reported in Figure 14 takes only on account cumulated returns and not their variance-covariance matrix. Therefore it is not affected by the fact that choosing best models on the basis of the single equation R^2 does not minimize the variance-covariance of the system residuals.

Lastly, we consider rates of returns at horizons of, respectively,1,12, and 24 months. The results are reported in Figure 15 and Table 4.

Insert Figures 15 and Table 4 here

In Table 4 we report for the three selected horizons the values of the average ex-post returns, of their variance, and the implicit levels of the CARA utility function, for a coefficient of risk aversion of 6. These quantities are derived for portfolio allocations based on thin modelling with the best R^2,on thin modelling with the worst R^2,on thick modelling(average portfolio allocation of the top fifty per cent model on the basis of their R^2), on the chosen benchmark and on thick modelling of returns generated by the minimization of the CARA utility function . The Table clearly shows a over-performance of thick modelling with respect to thin modelling, but not with respect to the benchmark. Lastly, minimization induces a sizeable increase in the variance of returns, confirming that recursive modelling can be useful for portfolio allocation of a risk-averse agent even if the sign tests reveal a very limited predictive accuracy for excess returns. The time-series used to construct Table 4 are reported in Figure 15.

7 Conclusions

The objective of this paper was the evaluation of the impact of predictability of asset returns in dynamic portfolio allocation with several assets in presence of model uncertainty. We have considered two asset classes, stock and bonds, and three geographic areas, US, Japan and Europe along with a safe return, Euro-denominated short-term interest rates. This gives a typical asset allocation problem, which can be evaluated using the typical benchmark chosen by investment funds. Our starting points are the results obtained in the context of recursive modeling for portfolio allocation among two assets (stock and cash for a single geographic area) proposed by Pesaran and Timmermann(2000). We consider explicitly model uncertainty by implementing thick modelling to consider the average portfolio allocation generated by the recursively selected top fifty per cent of models in term of adjusted R^2. The portfolio allocation based on this strategy leads to systematic over-performance with respect to optimal portfolio allocation among several assets based on the predictions of the best model as selected by the adjusted R^2 . Such over-performance is mainly attributable to a reduction in the volatility of the returns on the selected portfolios. However, thick modelling does not lead to over-performance with respect to a typical benchmark for our asset allocation problem.

A Data Appendix

All the data are at monthly frequencies (end of period observation) and are taken from DATASTREAM:

$P_t^{stock,EU}$	TOTMKBD~DM"	Germany -DS market- Price Index DM
$P_t^{stock,US}$	TOTMKUS~DM	US -DS market- Price Index DM
$P_t^{stock,JP}$	TOTMKJP~DM	Japan -DS market- Price Index DM
$P_t^{stock,EU}$	TOTMKEM~DM	EMU -DS market- Price Index DM
dy_t^{US}	TOTMKUS(DY)	US -DS market- Dividend yield
dy_t^{JP}	TOTMKJP(DY)	Japan -DS market- Dividend yield
dy_t^{EU}	TOTMKBD(DY)	Germany -DS market- Dividend yield
dy_t^{EU}	TOTMKEU(DY)	EU -DS market- Dividend yield
pe_t^{US}	TOTMKUS(PE)	US-DS MARKET - PER
pe_t^{JP}	TOTMKJP(PE)	JAPAN-DS MARKET - PER
pe_t^{EU}	TOTMKEU(PE)	EU-DS MARKET - PER
pe_t^{EU}	TOTMKBD(PE)	GERMANY-DS MARKET - PER
$P_t^{bond,US}$	BMUS07Y(RI)	US BENCHMARK 7-Y DS GOVT. INDEX
$P_t^{bond,EU}$	BMBD07Y(RI)	BD BENCHMARK 7-Y DS GOVT. INDEX
$P_t^{bond,JP}$	AJPGVG4(RI)	JP TOTAL 7-10 Y DS GOVT. INDEX
$R_t^{US,c}$	LHIBAAL(RY)	LEHMANN LONG BAA CORP. INDEX
R_t^{US}	BMUS10Y(RY)	US BENCHMARK 10 Y DS GOVT. INDEX
$R_t^{JP,c}$	JPYIB10Y	JP GOVERNMENT BOND YIELD
R_t^{EU}	BMBD10Y(RY)	BD BENCHMARK 10 Y DS GOVT. INDEX
R_t^{JP}	JPNKBNDLF	JP NIKKEI BOND INDEX YIELD - LONG-TERM (EP)
$R_t^{EU,c}$	BDWZ9826	BD LONG CORPORATE INDEX
r_t^{US}	ECUSD3M	US EURO-$ 3 MONTH
r_t^{JP}	ECJAP3M	JAPAN EURO-YEN 3 MONTH
r_t^{EU}	ECWGM3M	GERMANY EURO-MARK 3 MTH
bc_t^{US}	USNAPMPR	US NAPM MAN. SURVEY: SADJ
bc_t^{JP}	JPLEADIN	JP LEADING DIFFUSION INDEX
bc_t^{EU}	BDIFOMTHF	BD MAN. TRADE: PRODN. EXP. - NEXT 3 MONTHS NAD
bc_t^{EU}	EMEUSIPAQ	EM IND. SURVEY: PROD.EXP. FOR MTH.AHEAD SADJ
ip_t^{US}	USINPRODG	US INDUSTRIAL PRODUCTION
ip_t^{JP}	JPI66..IF	JP INDUSTRIAL PRODUCTION
ip_t^{EU}	BDINPRODG	BD INDUSTRIAL PRODUCTION
ip_t^{EU}	EMESINPRG	EM INDUSTRIAL PRODUCTION

rs_t^{US}	USRETTOTD	US RETAIL SALES
rs_t^{JP}	JPOCRSALG	JP RETAIL SALES
rs_t^{EU}	BDOCRSALG	BD RETAIL SALES
rs_t^{EU}	EMESRETSG	EM RETAIL SALES
cc_t^{US}	USCNCONF	US CONSUMER CONFIDENCE INDEX SADJ
cc_t^{JP}	JPEPACONQ	JP CONSUMER CONFIDENCE
cc_t^{EU}	BDEUSCCIQ	BD CONSUMER CONFIDENCE INDEX SADJ
cc_t^{EU}	EMEUSCCIQ	EM CONSUMER CONFIDENCE INDEX SADJ
$M3_t^{US}$	USOCM3MNB	US MONEY SUPPLY - M3
$M3_t^{JP}$	JPLEMQMPE	JP MONEY SUPPLY - M2+CD
$M3_t^{EU}$	EMECBM3.B	EM MONEY SUPPLY: M3
$M3_t^{EU}$	BDOCM3MNB	BD MONEY SUPPLY - M3
cpm_t	ECINFA$	ECONOMIST COMMODITY PRICE INDEX
$demusd_t$	UKXDMK../UKXUS$..	US Dollar/Deutschemark ex.rate.
$yenusd_t$	UKXYEN../UKXUS$..	
oil_t	WDI76AAZA	WD MARKET PRICE-PETROLEUM,SPOT

References

Barberis N.(2000) 'Investing for the Long run when Returns are Predictable', *Journal of Finance*, 55, 1, 225-264

Campbell J.Y. and R.Shiller (1987) 'Cointegration and tests of present value models" *Journal of Political Economy*,95, 1062-1088.

Campbell J.Y. and R.Shiller (1988a) 'The dividend-price ratio and expectations of future dividends and discount factors', *Review of Financial Studies*, 1, 195-227.

Campbell J.Y. and R.Shiller (1988b) 'Stock prices, earnings, and expected dividends', *Journal of Finance*, 43, 661-676.

Campbell, J. Lo, A. and McKinlay (1997) *The Econometrics of Financial Markets* Princeton University Press

Campbell J.Y. and L.Viceira (1999) 'Consumption and portfolio decisions when expected returns are time-varying', *Quarterly Journal of Economic*s, 114, 433-495.

Cochrane J.(1999) 'Portfolio advice for a multifactor world', NBER wp 7170

Doppelhofer G., Miller R.I.,Sala-i-Martin(2000) 'Determinants of long-term growth:a Bayesian averaging of classical estimates(BACE) approach', NBER wp 7750

Granger C.W.J.(2000) 'Thick modelling', UCSD, mimeo

Granger C.W.J. and A.Timmermann (1999) 'Data mining with local model specification uncertainty: a discussion of Hoover and Perez', *The Econometrics Journal*, 2, 220-226

Hoeting J., D.Madigan, A.Raftery and C.Volinsky (1999) 'Bayesian Model Averaging: a Tutorial' Technical Report 9814, Department of Statistics, Colorado State University

Kandel S., and R.S. Stambaugh (1996) 'On the Predictability of Stock Returns: An Asset-Allocation Perspective', *Journal of Finance*, 51, 2, 385-424

Keim D., and R.Stambaugh (1986) 'Predicting returns in the stock and bond markets' *Journal of Financial Economics*, 17, 357-390

Lander J., Orphanides A. and M.Douvogiannis (1997) "Earning forecasts and the predictability of stock returns: evidence from trading the S&P" Board of Governors of the Federal Reserve System, http:\\www.bog.frb.fed.org

Lamont O.(1998) 'Earnings and expected returns' *Journal of Finance*, 53, 5, 1563-1587

Pesaran M.H. and A.Timmermann (2000) 'A Recursive Modelling Approach to Predicting UK Stock Returns', *The Economic Journal*, forthcoming

Pesaran M.H. and A.Timmermann (1995) 'Predictability of Stock Returns: Robustness and Economic Significance', *Journal of Finance*, 50, 4, 1201-1228

Pesaran M.H. and A. Timmermann (1992) 'A simple non-parametric test of predictive performance', *Journal of Business and Economics Statistics*, 10, 461-465

Raftery A., Madigan D. and J.Hoeting(1997) 'Bayesian model averaging for linear regression models' *Journal of the American Statistical Association*, 92(437), 179-191

Samuelson P.A.(1969) 'Lifetime portfolio selection by dynamic stochastic programming', *Review of Economics and Statistics, 51, 239-246.*

Siegel, J. (1994) *Stocks for the Long Run*, Richard D.Irwin, Burr Ridge, III.

Sullivan R., A.Timmermann and H.White (1999) 'Data-snooping, technical trading rules performance and the bootstrap', *Journal of Finance*, 54, 1647-1692

Table 1: Data description

	$ER_t^{stock,US}$	$ER_t^{stock,JP}$	$ER_t^{stock,EU}$	$ER_t^{bond,US}$	$ER_t^{bond,JP}$	$ER_t^{bond,EU}$
Mean	0.994295	0.533552	0.813405	0.258189	0.484891	0.196261
Median	1.209115	0.478314	1.106681	0.155691	0.311222	0.410888
Max	14.37010	22.32307	13.27408	11.22538	14.99584	2.611280
Min	26.12715	-18.74755	-22.04453	-9.104957	-7.824083	-4.191668
Std. Dev.	5.797650	7.215082	5.325057	3.514238	3.625911	1.205883
Skewness	-0.603130	0.157378	-0.738908	0.384001	0.492237	-0.609543
Kurtosis	4.723641	3.248839	5.246071	3.379549	3.846602	3.161211
Jarque-Bera	37.25219	1.355014	60.84217	6.176855	14.18987	12.72734
Probability	0.000000	0.507882	0.000000	0.045574	0.000829	0.001723
Observations	202	202	202	202	202	202

Correlation matrix of excess returns.

Sample: $1984:1-2000:10$
$(1984{:}1-1991{:}12, 1992{:}1-2000{:}10)$

	$ER_t^{stock,US}$	$ER_t^{stock,JP}$	$ER_t^{stock,EU}$	$ER_t^{bond,US}$	$ER_t^{bond,JP}$	$ER_t^{bond,EU}$
$ER_t^{stock,US}$	1	0.40 (0.38,0.44)	0.52 (0.42,0.65)	0.71 (0.72,0.68)	0.23 (0.17,0.30)	0.08 (-0.09,0.27)
$ER_t^{stock,JP}$		1	0.32 (0.33,0.32)	0.22 (0.23,0.22)	0.54 (0.66,0.42)	0.12 (0.21,0.04)
$ER_t^{stock,EU}$			1	0.28 (0.18,0.40)	0.05 (-0.004,0.11)	0.19 (0.19,0.18)
$ER_t^{bond,US}$				1	0.28 (0.23,0.33)	0.17 (-0.08,0.46)
$ER_t^{bond,JP}$					1	0.31 (0.32,0.29)
$ER_t^{bond,EU}$						1

Table 2: The specification of forecasting equations for excess returns

Focal variables, A_t		
	Shares	Bonds
US	$\left\{c, pe_t^{US}, R_t^{US}, r_{t,t+1}^{US}\right\}$	$\left\{c, R_t^{US}, r_{t,t+1}^{US}, r_{t-1,t}^{US}\right\}$
JP	$\left\{c, pe_t^{JP}, R_t^{JP}, r_{t,t+1}^{JP}\right\}$	$\left\{c, R_t^{JP}, r_{t,t+1}^{JP}, r_{t-1,t}^{JP}\right\}$
EU	$\left\{c, pe_t^{EU}, R_t^{EU}, r_{t,t+1}^{EU}\right\}$	$\left\{c, R_t^{EU}, r_{t,t+1}^{EU}, r_{t-1,t}^{EU}\right\}$

Semi-focal variables, B_t		
	Shares	Bonds
US	$\left\{\begin{array}{c} r_{t,t+1}^{US}, dy_t^{US}, sev_t^{US}, sp_t^{US}, \\ \Delta_{12}lip_{t-1}^{US}, r_{t,t+1}^{EU}, \Delta_{12}m3_{t-1}^{U}, \\ \Delta_{12}cpm_t, demusd_t, \Delta_{12}oil_t \end{array}\right\}$	$\left\{\begin{array}{c} r_{t,t+1}^{EU}, sev_t^{b,US}, sev_t^{s,US}, sp_t^{US}, \\ \Delta_{12}lip_{t-1}^{US}, \Delta lrs_{t-1}^{US}, \Delta_{12}m3_{t-1}^{US}, \\ \Delta_{12}cpm_t, \Delta_{12}oil_t, demusd_t \end{array}\right\}$
JP	$\left\{\begin{array}{c} r_{t,t+1}^{US}, r_{t,t+1}^{EU}, dy_t^{JP}, sev_t^{JP}, \\ sev_t^{US}, sp_t^{JP}, \Delta_{12}lip_{t-1}^{JP}, \Delta_{12}m3_{t-1}^{JP}, \\ \Delta_{12}cpm_t, yendem_t \end{array}\right\}$	$\left\{\begin{array}{c} r_{t,t+1}^{EU}, sev_t^{b,JP}, sev_t^{b,US}, sev_t^{s,US}, \\ sp_t^{JP}, \Delta_{12}lip_{t-1}^{JP}, \Delta_{12}m3_{t-1}^{JP}, \\ \Delta_{12}cpm_t, yendem_t, demusd_t \end{array}\right\}$
EU	$\left\{\begin{array}{c} r_{t,t+1}^{US}, r_{t-1,t}^{EU}, dy_t^{EU}, sev_t^{EU}, \\ sp_t^{EU}, \Delta_{12}lip_{t-2}^{EU}, cc_{t-1}^{EU}, \\ \Delta_{12}cpm_t, yenusd_t, demusd_t \end{array}\right\}$	$\left\{\begin{array}{c} r_{t,t+1}^{US}, sev_t^{b,EU}, sev_t^{b,US}, sp_t^{EU}, \\ \Delta_{12}lip_{t-2}^{EU}, \Delta lrs_{t-2}^{EU}, cc_{t-1}^{EU}, \\ \Delta_{12}cpm_t, demusd_t, yenusd_t \end{array}\right\}$

R_t^i, is the yield to maturity of long term bonds
$r_{t,t+1}^i$is the short term interes rate
pe^i is the log of the price earning ratio
sp_t^i is the ratio of corporate bond yields on government bond yields
$sev_t^{i,j}$ is a measure of the volatility on asset i in country j
$\Delta_{12}lip_t^i$ is the annual rate of change in the index of industrial production
Δlrs_t^i is the monthly change in the index of retail sales
$\Delta_{12}m3_t^i$ is the annual rate of change of the money supply
$\Delta_{12}cpi_t^i$, is the annual rate of change of retail prices
$\Delta_{12}cpm_t^i$ is the rate of change of commodity price index
$\Delta_{12}oil_t$ is the rate of change of the spot price of oil
cc_t^i is the level of consumer confidence index
bc_t^i is the level of business confidence index
$demusd_t$ is the log of DEM/USD exchange rate
$yendem_t$ is the log of YEN/DEM exchange rate
$yenusd_t$ is the log of YEN/USD exchange rate
NB the suffix i always denotes the area, where i=US,JP, EU

Table 3: Sign test of excess returns in stock and bonds markets

		1990-2000	1990-1995	1995-2000
$ER_t^{stock,US}$				
Best R^2				
	PT-statistic	-1.969	-3.544	0.080
	Proportion of Correct Signs %	0.4538	0.2623	0.6143
Worst R^2				
	PT-statistic	0.1484	0.640	-1.191
	Proportion of Correct Signs %	0.5077	0.4262	0.5714
Average				
	PT-statistic	-0.4612	-1.4260	-0.2404
	Proportion of Correct Signs %	0.50	0.3934	0.5857
$ER_t^{stock,JP}$				
Best R^2				
	PT-statistic	0.9237	0.6058	0.3265
	Proportion of Correct Signs %	0.5385	0.5410	0.5286
Worst R^2				
	PT-statistic	1.7565	2.3288	0.1873
	Proportion of Correct Signs %	0.5769	0.6557	0.5143
Average				
	PT-statistic	1.4415	0.7814	0.8460
	Proportion of Correct Signs %	0.5615	0.5574	0.5571
$ER_t^{stock,EU}$				
Best R^2				
	PT-statistic	3.2404	2.4252	1.5186
	Proportion of Correct Signs %	0.5923	0.5410	0.6429
Worst R^2				
	PT-statistic	2.1415	0.2708	1.8637
	Proportion of Correct Signs %	0.6077	0.5246	0.6857
Average				
	PT-statistic	1.6513	0.5860	0.6928
	Proportion of Correct Signs %	0.5923	0.5410	0.6429

Table 3: continued

		1990-2000	1990-1995	1995-2000
$ER_t^{bond,US}$				
Best R^2				
	PT-statistic	-0.123	-0.791	-0.070
	Proportion of Correct Signs %	0.492	0.475	0.5
Worst R^2				
	PT-statistic	-1.07	-1.914	-0.846
	Proportion of Correct Signs %	0.454	0.410	0.486
Average				
	PT-statistic	-0.844	-2.197	-0.286
	Proportion of Correct Signs %	0.462	0.410	0.500
$ER_t^{bond,JP}$				
Best R^2				
	PT-statistic	-1.567	-0.665	-1.755
	Proportion of Correct Signs %	0.431	0.459	0.400
Worst R^2				
	PT-statistic	-0.025	-0.411	0.542
	Proportion of Correct Signs %	0.492	0.475	0.514
Average				
	PT-statistic	-0.258	-0.136	-0.297
	Proportion of Correct Signs %	0.485	0.492	0.471
$ER_t^{bond,EU}$				
Best R^2				
	PT-statistic	1.140	-0.088	1.513
	Proportion of Correct Signs %	0.531	0.475	0.571
Worst R^2				
	PT-statistic	1.772	0.449	1.950
	Proportion of Correct Signs %	0.577	0.508	0.643
Average				
	PT-statistic	1.635	0.665	1.391
	Proportion of Correct Signs %	0.569	0.525	0.600

The PT-statistic is the Pesaran-Timmerman non-parametric test of predictive performance. Let $x_t = E(y_t, \Omega_{t-1})$ be the predictor of y_t found with respect to the information set, Ω_{t-1} , with n observations $(y_1, x_1), (y_2, x_2), \ldots, (y_n, x_n)$ available. The test proposed by Pesaran and Timmerman (1992) is based on the proportion of times that the direction of changes in y_t is correctly predicted by x_t. The test statistic is computed as

$$Sn = \frac{P - P^*}{\{V(P) - V(P^*)\}^{1/2}} \sim N(0,1) \tag{5}$$

where:

$$\begin{aligned}
P &= \bar{Z} = \frac{1}{n}\sum_{i=1}^{n} Z_i \\
P^* &= P_y P_x + (1 - P_y)(1 - P_x) \\
V(P^*) &= \frac{1}{n} P^* (1 - P^*) \\
V(P) &= n \begin{pmatrix} (2P_y - 1)^2 P_x (1 - P_x) + (2P_x - 1)^2 P_y (1 - P_y) + \\ + \frac{4}{n} P_y P_x (1 - P_y)(1 - P_x) \end{pmatrix}
\end{aligned}$$

Z_i is an indicator variable which takes value of one when the sign of y_t is correctly predicted by x_t, and zero otherwise, P_y is the proportion of times y_t takes a positive value, P_x is the proportion of times x_t takes a positive value.

Table 4: performance

Model	Horizon in months	Mean	Variance	Utility Function
Best R^2	1	0.0125	0.0022	0.0059
Worst R^2	1	0.0129	0.0017	0.0077
thick	1	0.0124	0.0017	0.0074
Benchmark	1	0.0116	0.0007	0.0093
thick (min)	1	0.0114	0.0022	0.0050
Best R^2	12	0.1416	0.0182	0.0871
Worst R^2	12	0.1545	0.0321	0.0582
thick	12	0.1492	0.0183	0.0942
Benchmark	12	0.1452	0.0123	0.1082
thick (min)	12	0.1606	0.0538	-0.0009
Best R^2	24	0.2817	0.0335	0.1811
Worst R^2	24	0.2960	0.0569	0.1252
thick	24	0.2995	0.0293	0.2115
Benchmark	24	0.2982	0.0197	0.2391
thick (min)	24	0.2818	0.0397	0.1627

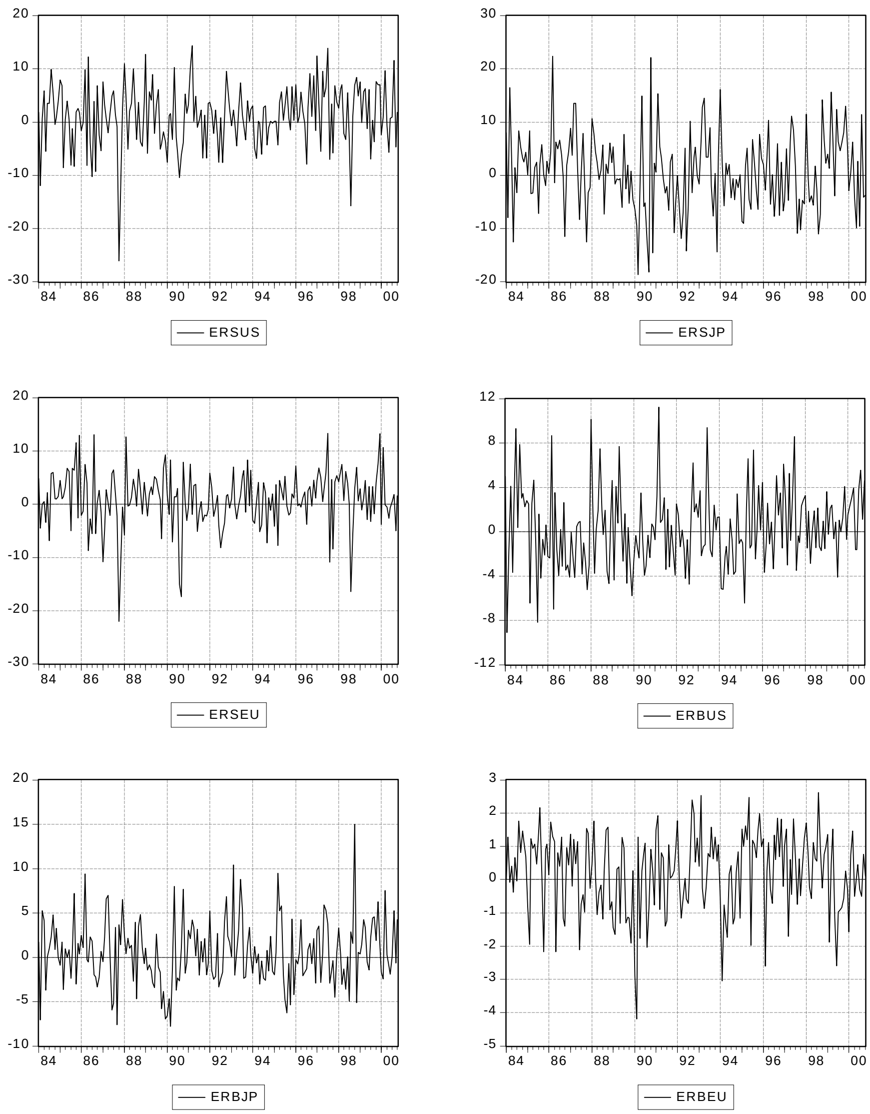


Figure 1: Monthly excess returns, with respect to a Euro-denominated short term interest rate, for two asset classes (stock and long-term bonds) and three areas (US, Europe and Japan)

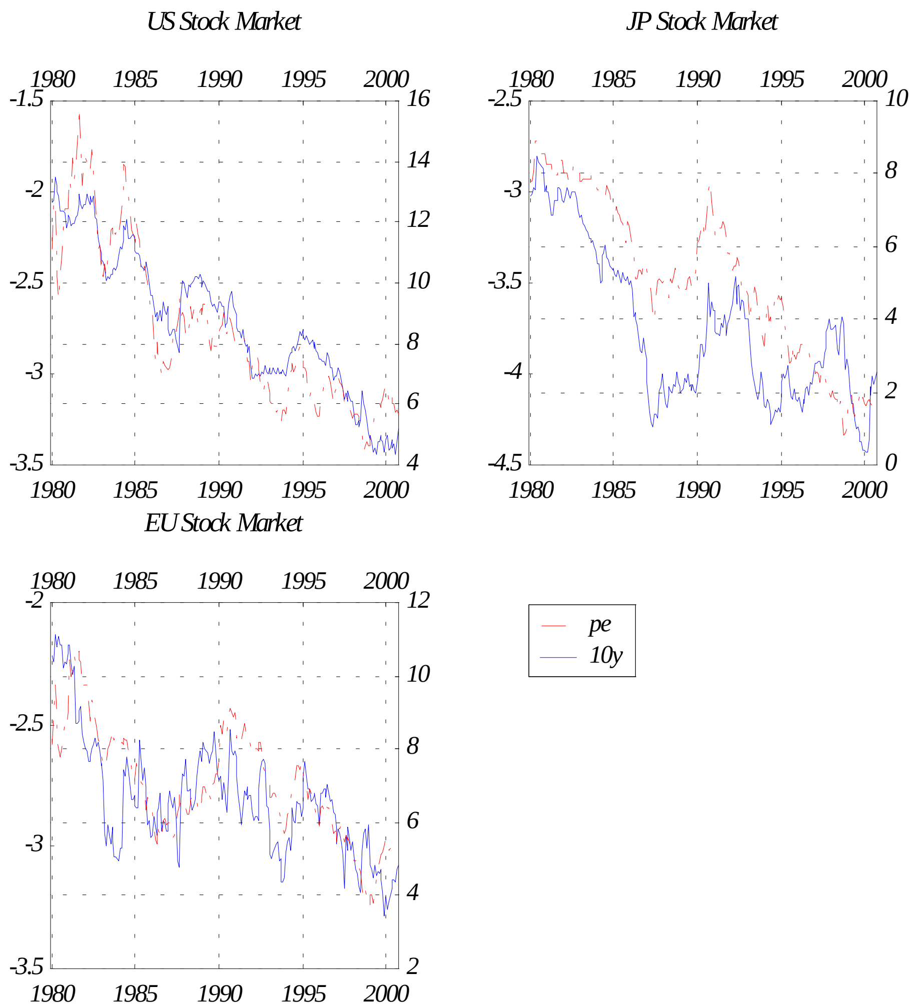


Figure 2: Long-run equilibria for the stock markets

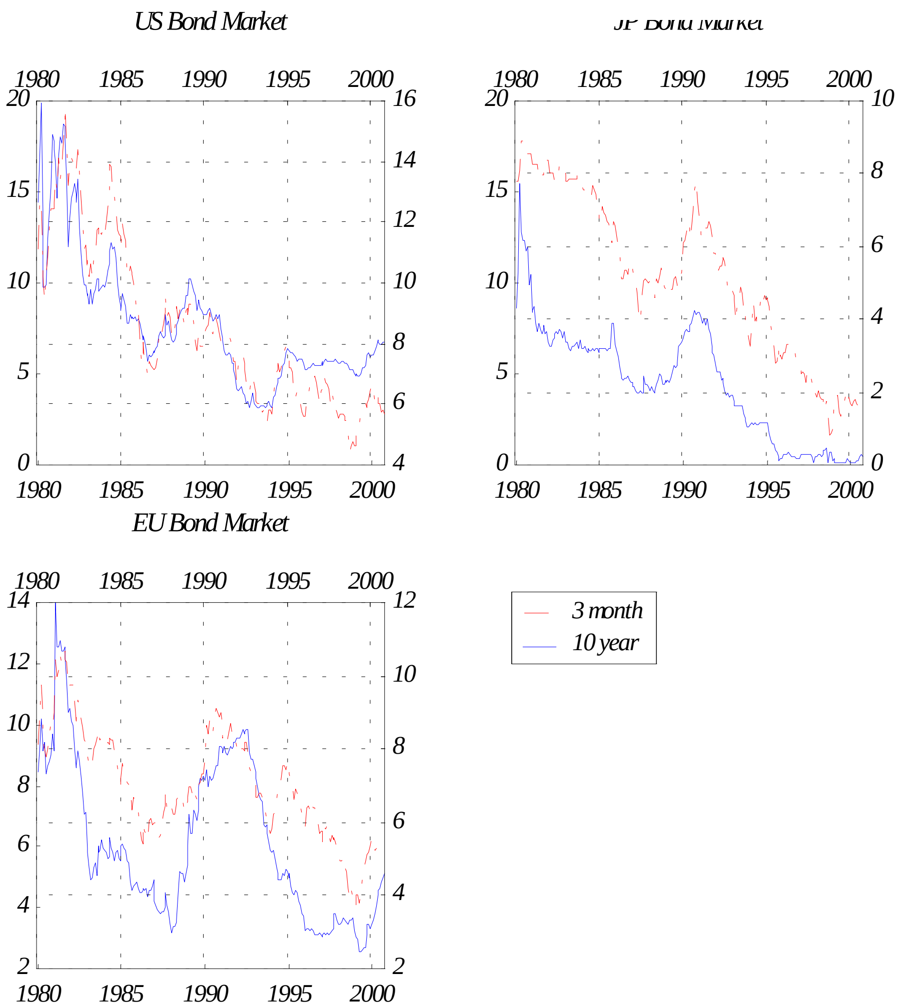


Figure 3: Long-run equilibria for the bond markets

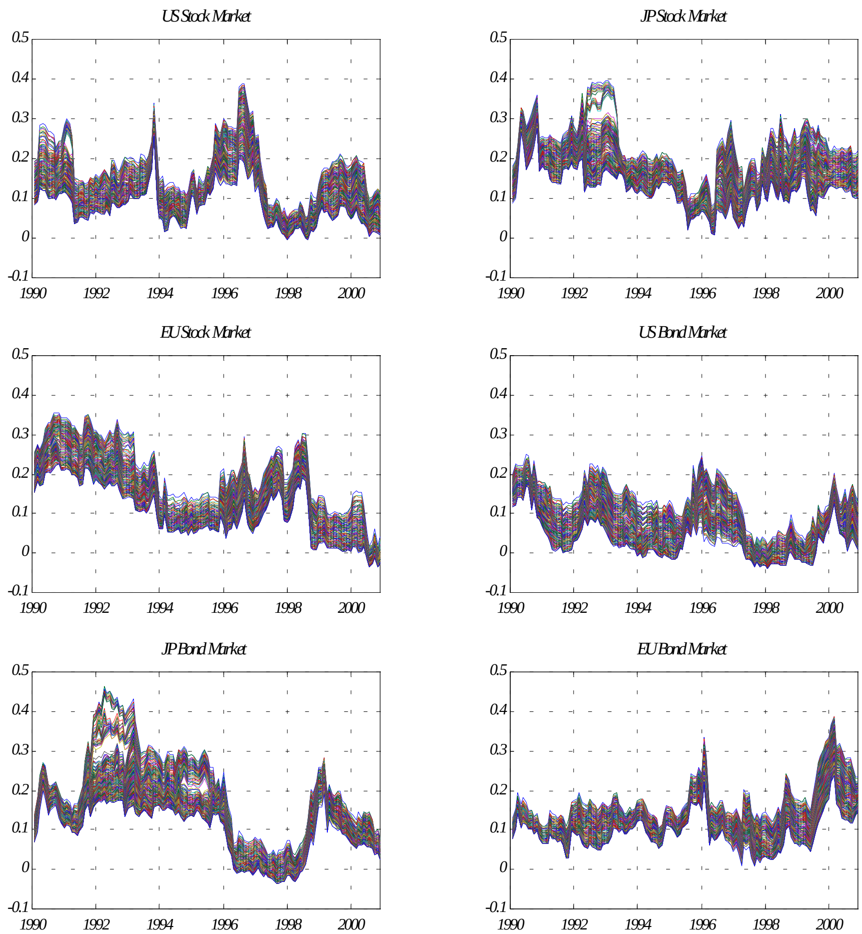


Figure 4: Adjusted R^2 for the top 50 per cent models of excess returns. Rolling regressions (size of the window: five years of monthly observations)

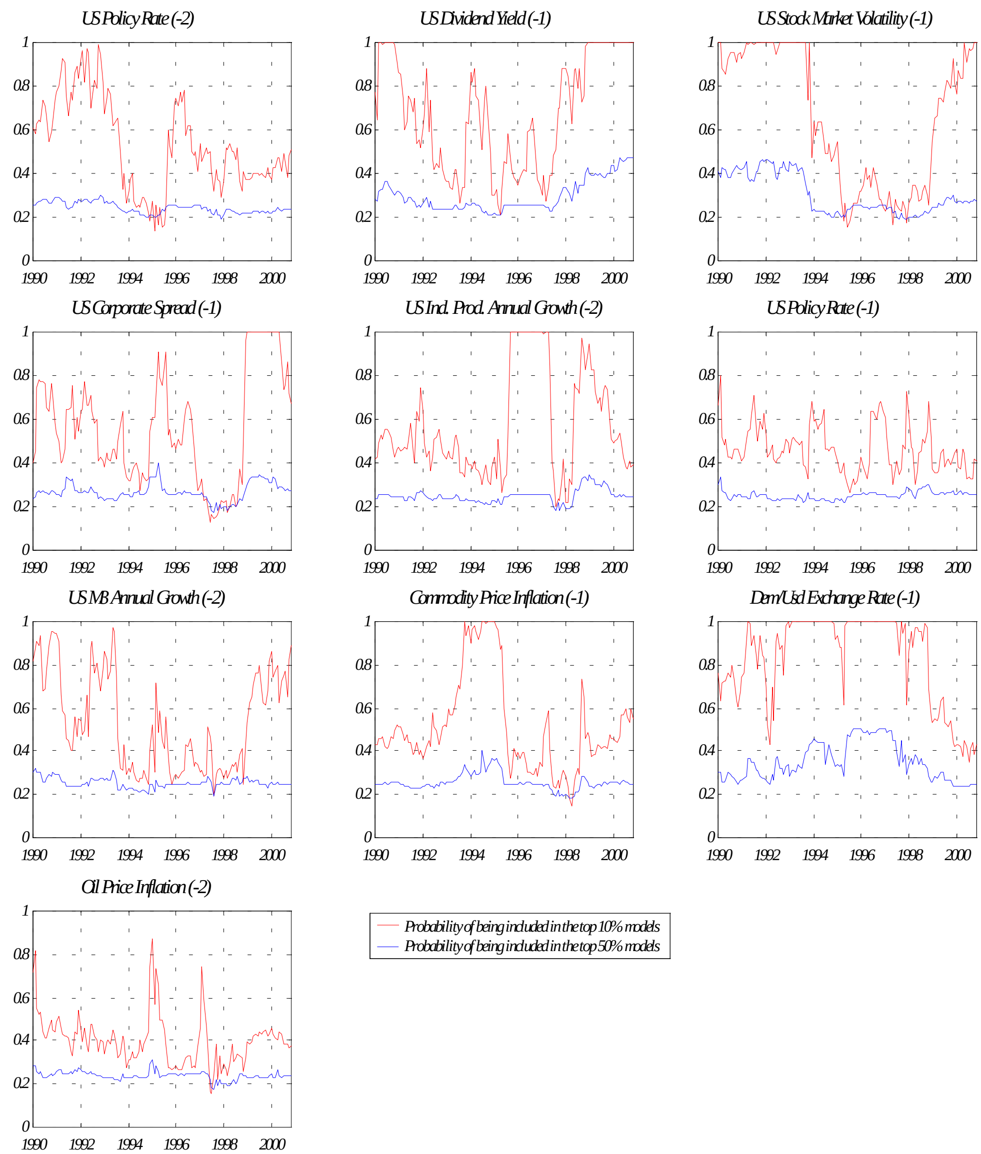


Figure 5: Modelling excess returns of US equities

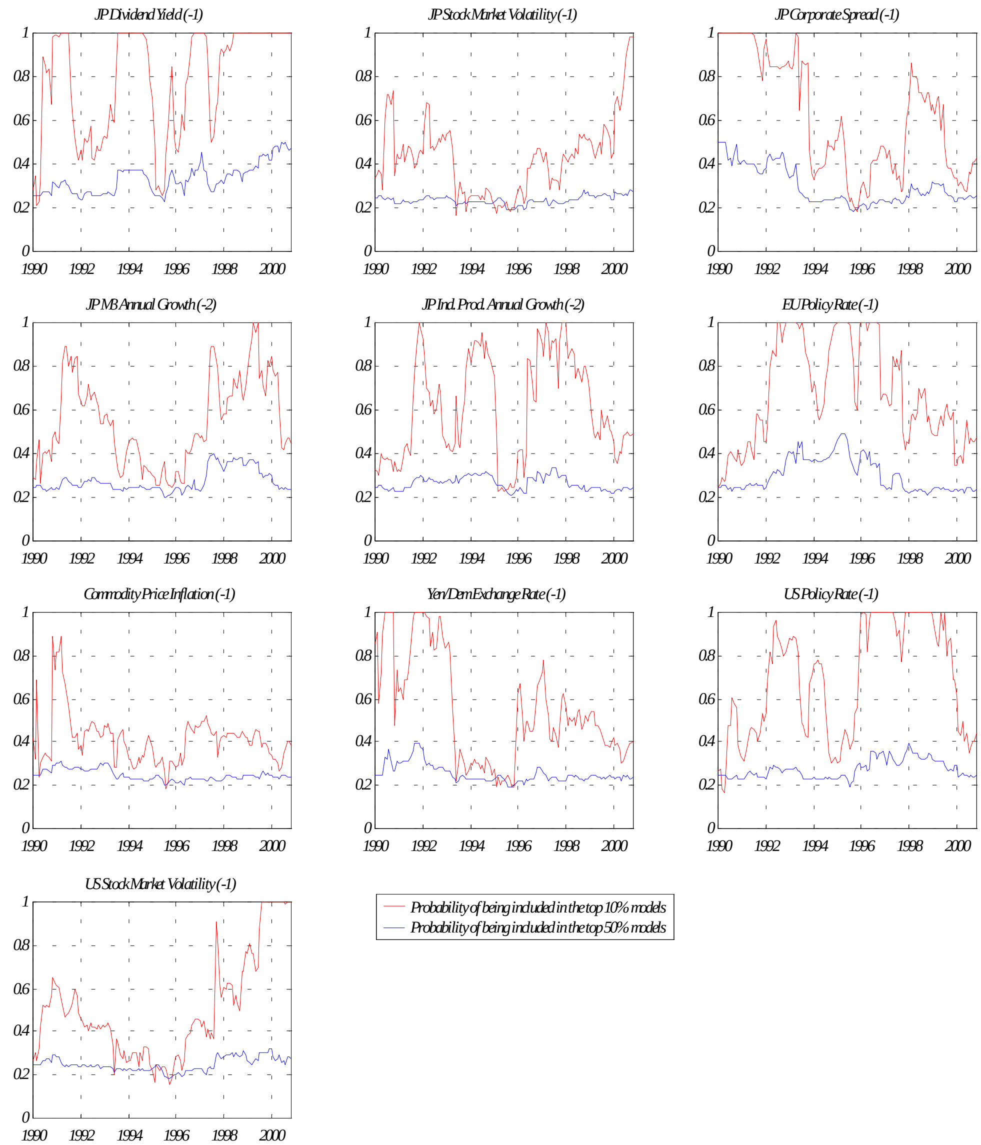


Figure 6: Modelling excess returns of Japan equities

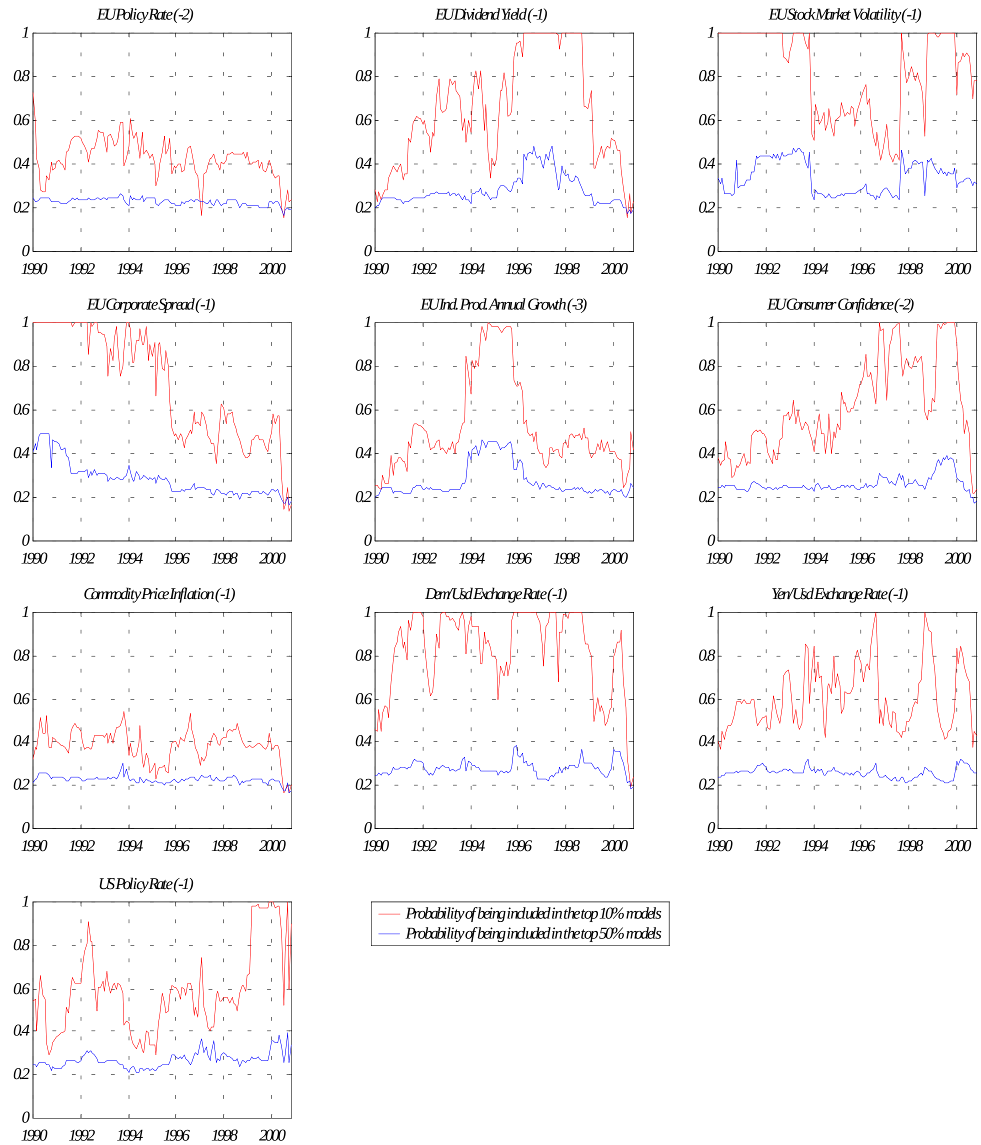


Figure 7: Modelling excess returns of European equities

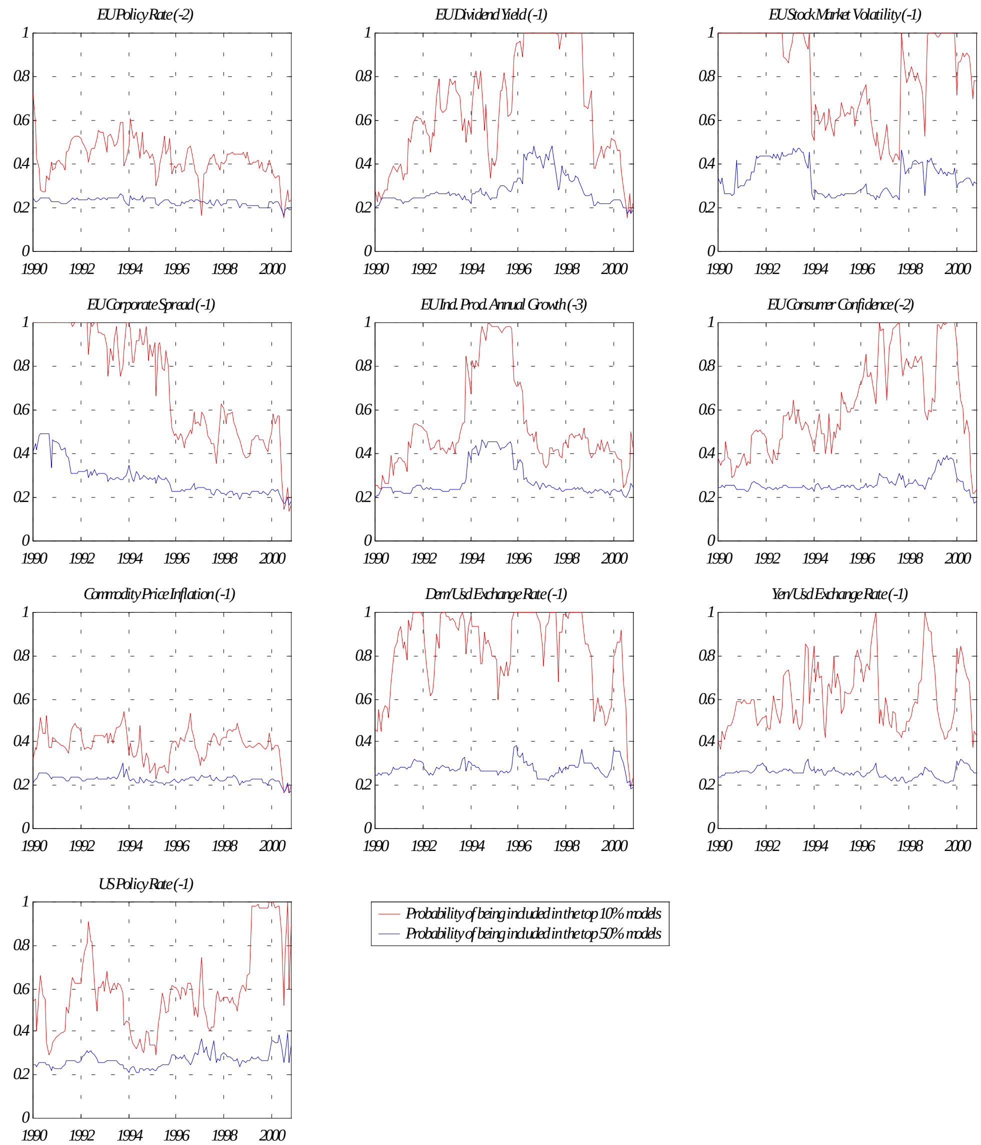


Figure 8: Modelling excess returns of US bonds

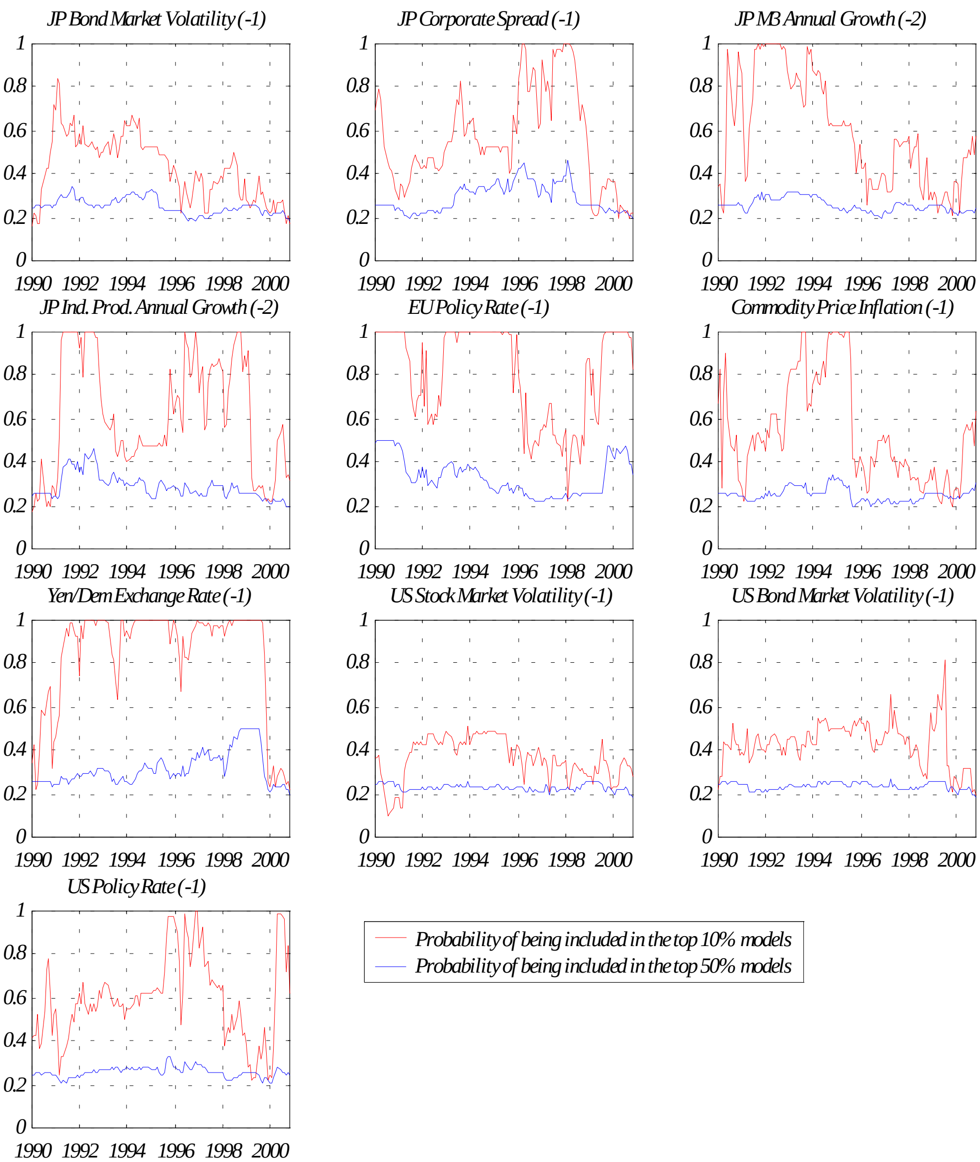


Figure 9: Modelling excess returns of Japan bonds

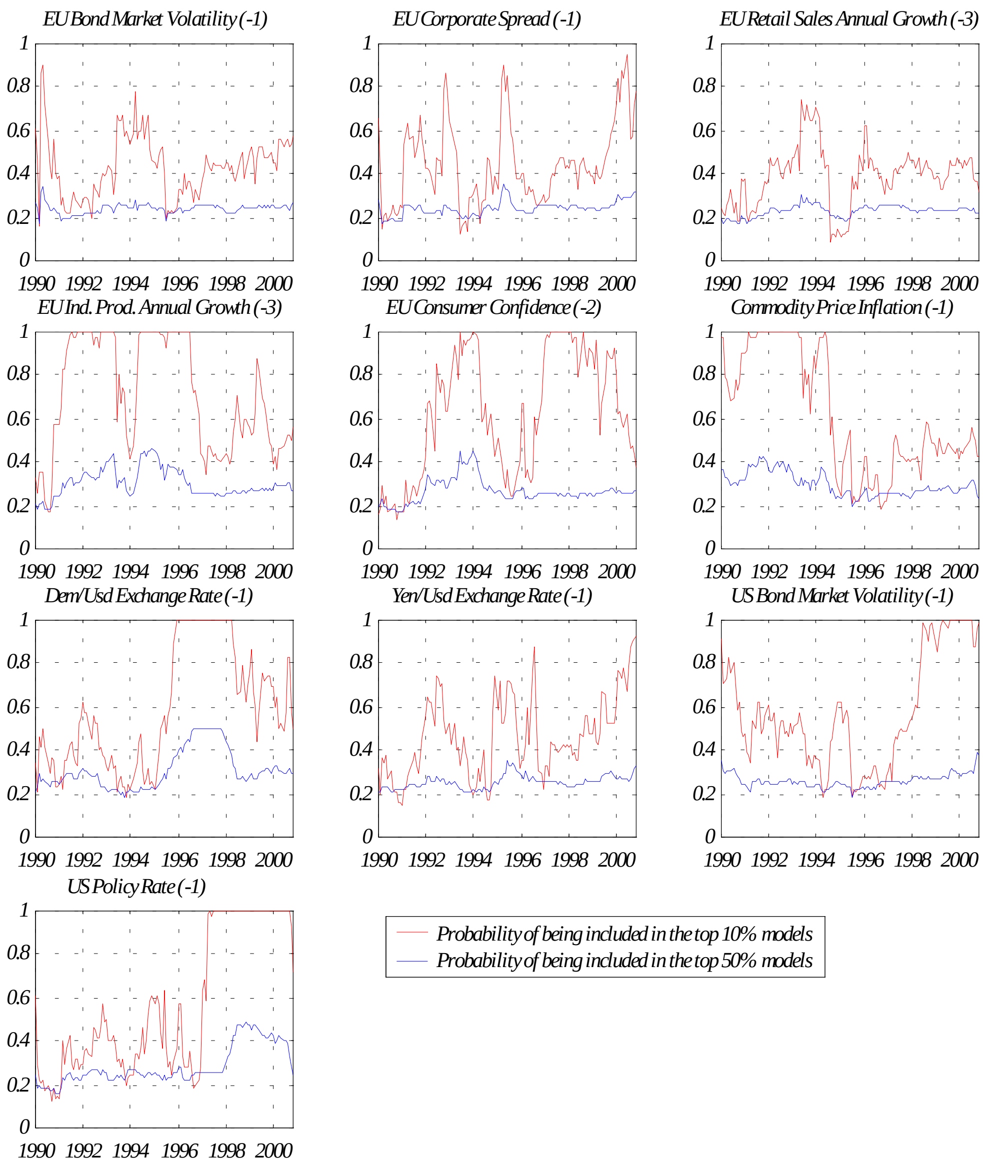


Figure 10: Modelling excess returns of Euro bonds

Figure 11: Weigths of Shares and Bonds with thick modelling

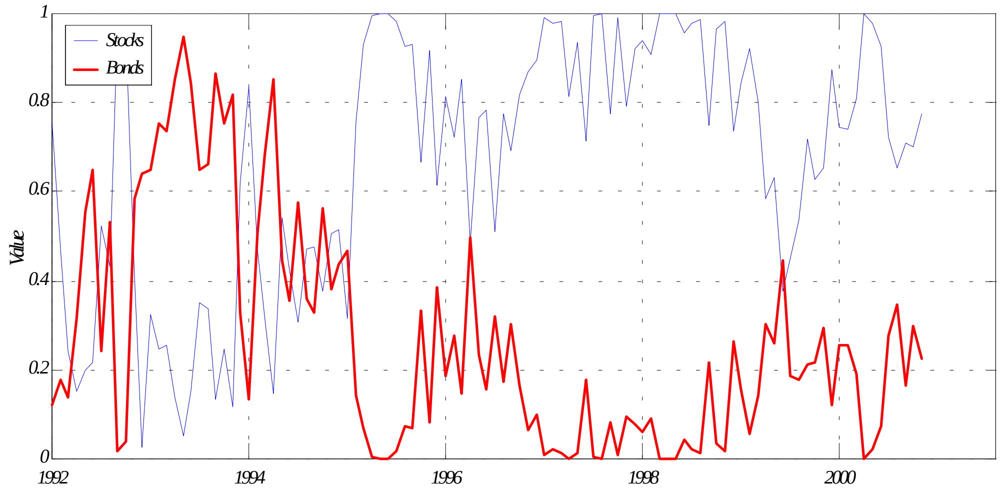
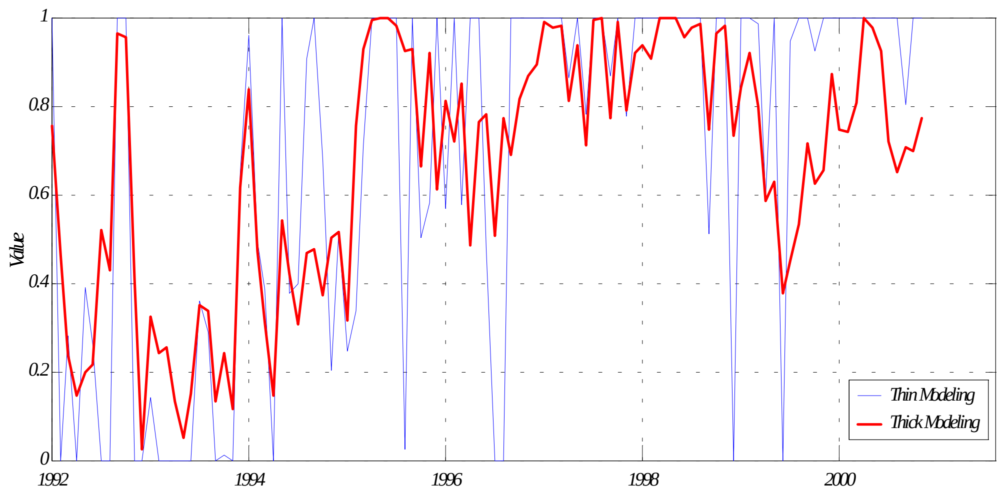


Figure 12: Weights of Shares with thin and thick Modelling



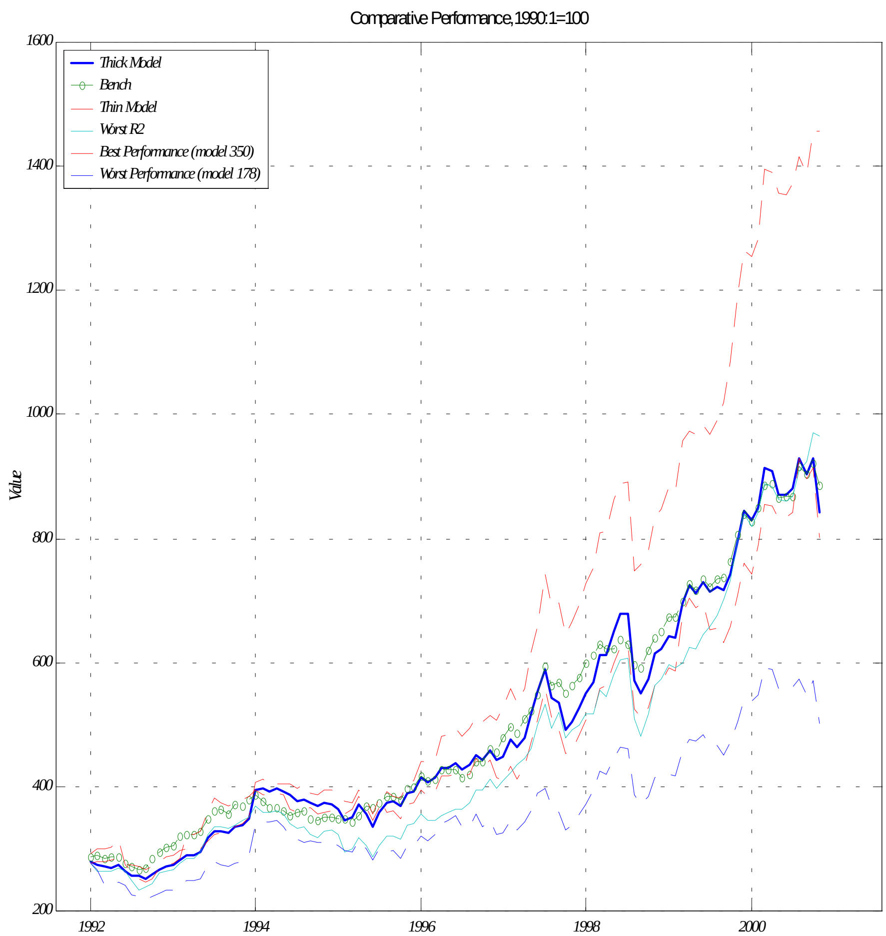


Figure 13: Comparative performance

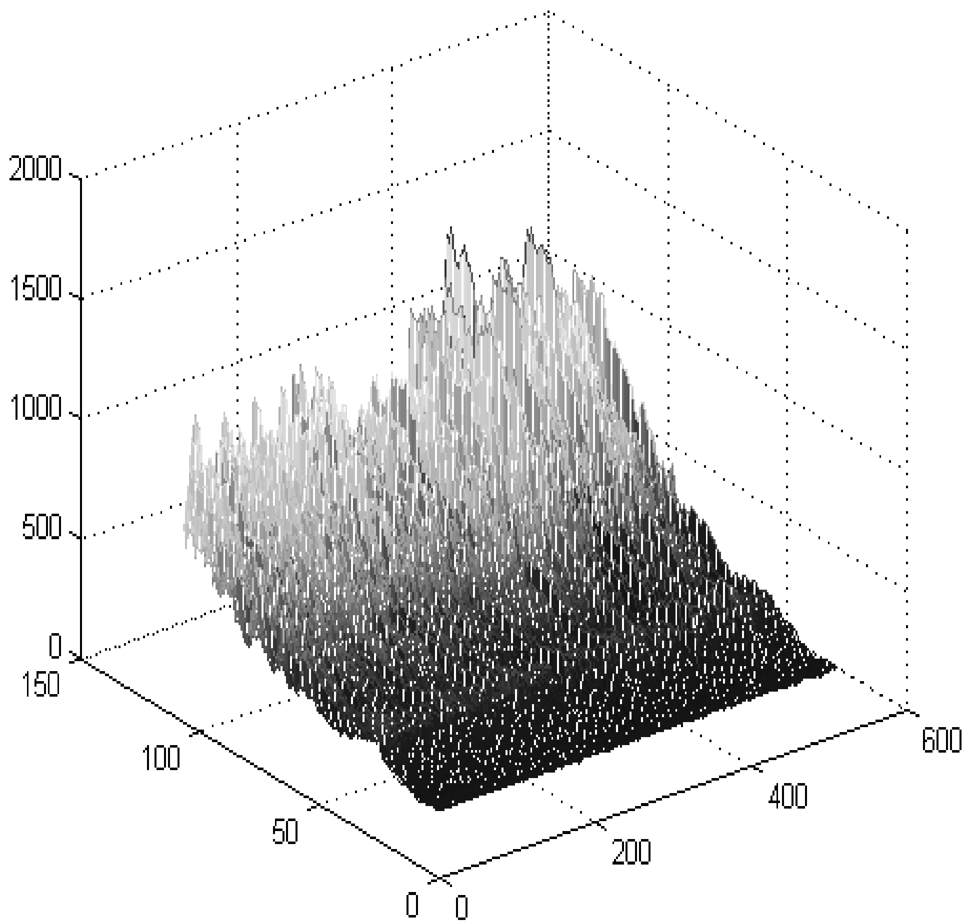


Figure 14: Comparative performance over time (obs 25 to 150) of predictive models ranked (from 1 to 512) by their adjusted R^2

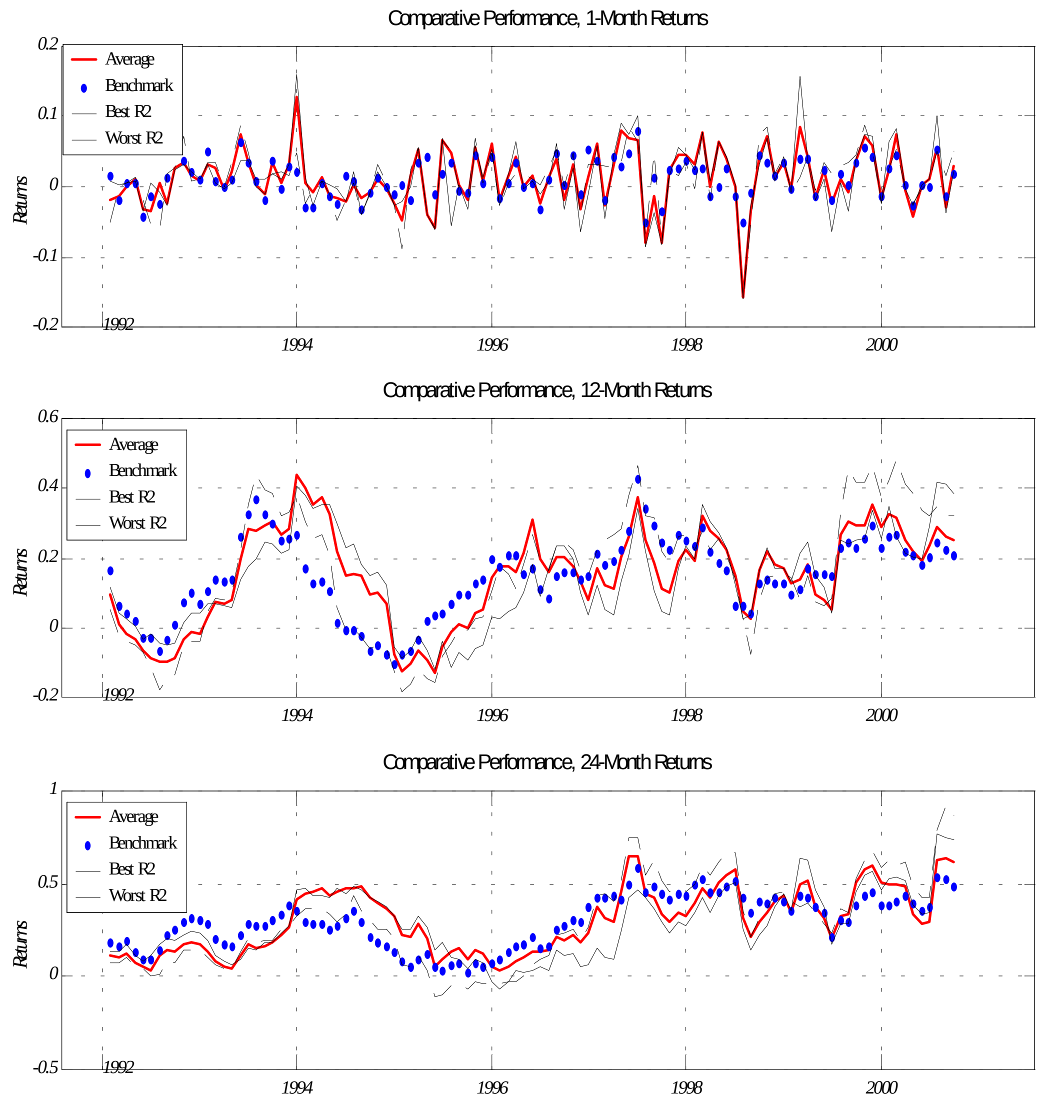


Figure 15: Period returns, from thick modelling, thin modeling with the best R^2, thin modelling with the worst R^2 and from the benchmark.